Loss Aversion Leading to Advantageous Selection

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Some insurance markets are characterized by "advantageous selection", that is, ex-post risk and coverage are negatively correlated. We show that expectation-based loss aversion as in Kőszegi and Rabin (2006, 2007) provides a natural explanation for this phenomenon when agents face modest-scale risks. More exposure to risk has two competing effects on an agent's willingness to pay for insurance: a positive effect, as in standard expected utility models; and a negative one, due to a reference effect. We determine conditions under which an insurance provider optimally sets a high price at which only low risk agents buy.

Classic economic theory predicts that ex-post risk and coverage are positively correlated in an insurance market (i.e., people that bought more insurance are more likely to have a bigger damage[1]); however, the opposite has been observed in a variety of settings. From the prospective of the insurer, this insurance puzzle is associated with *advantageous selection*: the agents who buy insurance suffer smaller damages than the agents who do not buy insurance. This implies that the insurer ends up covering the cheapest segment of the market.

Although not ubiquitous, evidence of advantageous selection has been found in several insurance markets. Examples include long term care insurance (Finkelstein and McGarry, 2006), car insurance (Chiappori et al., 2006; Saito, 2006), and credit card insurance (de Meza and Webb, 2001). Another interesting case is the large market of extended warranties.[2] A wide variety of goods (e.g., washing machines, refrigerators, cooking ranges, and other appliances) are usually sold bundled with base warranties. These are contracts that offer coverage for defects in materials and workmanship, power surge, as well as normal wear and tear. In other words, they protect the consumer against damages that are not subject to moral hazard. Extended warranties prolong the protection offered by base warranties for a given number of years.[3] According to consumer reports and product reviews,[4] extended warranties are priced relatively high compared to the value of

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[1]This is shown, for example, in models of pure adverse selection (Rothschild and Stiglitz, 1976; Chiappori et al., 2006; Bolton and Dewatripont, 2004), and in models of pure moral hazard (Arnott and Stiglitz, 1988).

[2]According to the Service Contract Industry Council, a trade group, 250 million extended warranties were sold in 2010 in the United States.

[3]Accidental Damage Warranty contracts are offered separately in some markets. These contracts offer coverage for damages like liquid spills or drops: typologies of accidents that are more exposed to moral hazard.

[4]See, for example, www.comsumerreports.org.

the products they cover, and are rarely used. These facts together are suggesting that advantageous selection may be occurring.

In this paper, we show that a behavioral model of expectation-based loss aversion provides a natural explanation for advantageous selection in markets of insurance products for small and modest-scale risks without moral hazard, like the one of extended warranties. In particular, we prove that the people that buy warranties may represent a biased sample of low risk individuals. Since Kahneman and Tversky (1979), loss aversion has been identified as a suitable framework to describe consumers' choices with respect to small and modest-scale risks (whereas the risk aversion framework often requires very high degrees of risk aversion to fit the observed behaviors).

We provide plausible conditions under which it is possible and optimal for the insurance provider to sell only to the lowest risk agents by increasing the price of her product. We consider an environment where the agents have the same degree of loss aversion, and the screening mechanism works only because of the reference dependent structure of the agents' preferences. In contrast to prior literature, we show that advantageous selection may arise in a model with minimal heterogeneity between the agents (agents differ only in terms of ex-ante risk) without moral hazard.

Our analysis unfolds in two stages: we first consider the agents' propensity to buy insurance; then we examine the strategic choice of the insurance provider in setting the price of her product. In each of these stages we unveil some novel results. First, in terms of the decision problem of a loss-averse agent, we extend and complement the analysis of Kőszegi and Rabin (2007) to characterize an agent's willingness to pay for an insurance product as a function of his risk exposure and the specification of his reference point; Kőszegi and Rabin (2007) consider how an agent chooses between gambles, but not his willingness to pay for a lottery. Second, with respect to pricing the insurance product, to the best of our knowledge we are the first to consider the strategic interaction between loss-averse agents and a profit-maximizing insurance provider. In our setting, both the price of the insurance and the final allocation (which agents — in terms of risk exposure — buy insurance) emerge as the equilibrium outcome of a strategic game. As a result, we are able to rationalize advantageous selection and predict in which environments advantageous selection is more likely to arise.

We consider a monopolist risk neutral insurance provider and loss averse agents with private information about their probability distributions over losses. Loss aversion implies that agents dislike losses more than they like equal-sized gains. In modeling each agent's decision problem, we adopt the framework of Kőszegi and Rabin (2007). Each agent's utility is represented as a sum of a consumption term and a gain-loss term. The gain-loss term increases or belittles the utility depending on how the consumption level compares to the reference point. The reference dependent preferences are expectation-based, that is, the reference point associated with any given action is provided by the consumption level induced by that action (as opposed, for example, to a generic status quo level). This is the central assumption in a number of theories (Bell, 1985; Loomes and Sugden, 1986; Gul, 1991; Kőszegi and Rabin, 2006), for which experimental evidence has been provided recently (Abeler et al., 2011; Ericson and Fuster, 2010).

Notice that, if an action is associated with a random outcome, a variety of potential reference points arises: it could be a specific consumption level, the expected value of the distribution, some other statistical function of an agent's random consumption level or the random consumption level itself.[5] In the decision problem we consider, the action of not buying insurance is associated with a random outcome, since the agent who does not buy remains exposed to the risk of incurring losses. In order to study the role of reference points, we focus on a general deterministic reference point[6] throughout the paper and consider stochastic reference points in the appendix.

Following Kőszegi and Rabin (2006, 2007), we use the concept of *preferred personal equilibrium* as the solution concept to study the decision problem of a loss averse agent. Informally, an action is a *personal equilibrium* if it maximizes the agent's utility when he expects to play this action, that is, when the reference point is the one associated with this action. Thus, the agent's reference points are determined endogenously as part of the equilibrium. When there is a multiplicity of personal equilibria, the *preferred personal equilibrium (PPE)* is the action that is associated with the highest ex ante expected utility.[7] We define an agent's *willingness to pay* for insurance as the maximum price at which buying insurance is a *PPE*.

We show that the distribution of losses an agent is exposed to may have two effects on his willingness to pay for insurance. On one hand, there is the direct effect of standard adverse selection, according to which higher losses increase an agent's willingness to pay. On the other hand, the distribution of losses may have an *indirect effect* on an agent's willingness to pay through the reference point.

When the indirect effect through the reference point is in the opposite direction of the direct effect of standard adverse selection and dominates, then advantageous selection may occur. In particular, if the reference point is "increasing" in the consumption distribution, then agents with larger loss distributions — and thus larger expected losses — associate lower reference points with the action of not buying insurance. This arises in a variety of settings (e.g., when the reference point is stochastic as in Kőszegi and Rabin (2006) or when it is given by the expected value of the consumption associated with an action). In general, the lower the reference point associated with not buying insurance, the less the agent

[5]For examples, see (Kőszegi and Rabin, 2006; Schmidt, Starmer and Sugden, 2008).

[6]A deterministic reference point is consistent with disappointment theory (Bell, 1985; Loomes and Sugden, 1986) and disappointment aversion (Gul, 1991), where the reference point is related to the expected value or the certainty equivalent. The mode (Bell, 1985) and the median (de Meza and Webb, 2007) have also been suggested as reasonable reference points in the literature. Furthermore, a number of empirical studies take the approach of a deterministic reference point (e.g., Crawford and Meng, 2011; Bohnet, Herrmann and Zeckhauser, 2010).

[7]We also consider an alternative solution concept proposed by Kőszegi and Rabin (2007): *Choice-acclimating Personal Equilibrium (CPE)*. However, in Appendix A we show that CPE may predict implausible behavior in our environment: an agent may buy insurance even if the price is greater than the loss associated with the worst case scenario.

is willing to pay for insurance.[8] Intuitively, an insurance product that provides a hedge against losses becomes less appealing to agents with lower reference points, because the cases in which the insurance allows them not to fall below their reference point (and suffer the extra disutility connected with the loss feeling) are more limited.

We analyze the strategic interaction between the insurance provider and the agents as a variant of the Stackelberg game: in the first stage, the insurance provider sets the price; in the second stage, each agent decides whether to buy insurance. We propose a novel solution concept to solve the game, *Stackelberg outcome in Preferred Personal Equilibria* (SPPE), which is a natural extension of sub-game perfect equilibrium to an environment where each player's best response is his *PPE* action. We show that advantageous selection arises in SPPE when the willingness to pay of the agents with lower expected loss is sufficiently high so that the seller obtains a higher profit from selling only to them. Even with minimal asymmetry across agents (same degree of loss aversion, and assuming that the reference point of each agent is given by the same statistic of the corresponding loss distribution), advantageous separation in the agents' choices may arise.

In addition to the loss distribution, an agent's reference point may also depend on several other factors. For instance, recent findings in the psychology literature identify *affection* (to a product) as a critical factor that influences the decision of buying insurance (Hsee and Kunreuther, 2000; Pham, 2007). Furthermore, the sense of guilt, regret, and pain associated with any event in which the good is damaged may also influence an agent's decision. In general, through the specification of the reference point in the utility function, it becomes possible to disentangle psychological factors from the individual's degree of risk or loss aversion. Our analysis offers an explanation for the inconsistencies in insurance purchasing decisions reported in various works both across different individuals and at a intra-personal level (Hsee and Kunreuther, 2000; Post et al., 2008).

The rest of the paper is organized as follows. In section I, we discuss related work. The model is set up in section II. We then introduce reference-dependent utilities and personal equilibria (section III). Section IV applies personal equilibria to insurance decisions. Section V derives properties of the willingness to pay for insurance. In section VI we derive conditions for advantageous selection, and in section VII we present some examples. In section VIII, we analyze advantageous selection in a setting with risk aversion and compare to the setting with loss aversion. Section IX concludes.

[8]The link between risk and reference points is applicable only when heterogeneity in the reference points is plausible. This seems more reasonable with modest-scale risk (e.g., damages in cars, appliances) than with high-scale risks. Nevertheless, advantageous selection also arises in the health insurance markets (a high-risk setting); and income effects have been used to justify it. On the other hand, income effects do not seem plausible justifications when considering modest-scale risks. Thus, it seems reasonable to distinguish the two situations and offer alternative explanations.

I. Related Work

Previous literature has tackled advantageous selection by providing theoretical explanations in contexts with moral hazard and heterogeneity in the degree of risk aversion (de Meza and Webb, 2001; Jullien, Salanié and Salanié, 2007; De Donder and Hindriks, 2009). In these papers, ex-ante all consumers are assumed to be exposed to the same probability distribution over losses, but in equilibrium, consumers who are more risk averse (ergo, more willing to buy an insurance product) are also the ones who spend more effort to minimize the probability of a damage. In general, all these explanations of advantageous selection based on the risk aversion framework entail both heterogeneity in risk aversion and a specific correlation between the degree of risk aversion and the level of risk each individual is exposed to: in order to have advantageous selection, the agent with high risk aversion needs also to be the one exposed to less risk. Moral hazard allows to endogenize the ex-interim (i.e., after deciding whether to buy insurance) distribution of risk over agents with different degrees of risk aversion in the desired way. However, moral hazard does not arise in certain insurance markets for which advantageous selection has been observed; e.g., this is the case for car insurance (Abbring, Chiappori and Pinquet, 2003). More importantly, several researchers (see Kőszegi and Rabin, 2007, for a list of references) have shown that the degree of risk aversion necessary to justify the observed insurance purchasing behavior with respect to small and modest scale risks is often extremely high and inconsistent with standard assumptions of diminishing-marginal utility of wealth.

Netzer and Scheuer (2010) consider a setting with heterogeneity in risk and patience and dynamic accumulation of wealth. The agents have ex-ante the same degree of risk-aversion, but, facing risk, each agent chooses between buying insurance or supplying labor, which leads to wealth accumulation. Under certain conditions, high-risk agents offer more labor and buy less insurance. Boone and Schottmuller (2011) use income to explain advantageous selection in health insurance markets by relying on the empirical observation that richer people tend to be healthier. Thus, the models of risk aversion that do not rely on moral hazard to explain advantageous selection use income effects instead. A setting with risk aversion and income effects may represent a plausible explanation for advantageous selection in insurance markets involving high-stake risks, but not for modest-scale risks.

A behavioral explanation for advantageous selection is provided by Huang, Muermann and Tzeng (2008). They consider a moral hazard setting, where the population is homogeneous in terms of ex-ante risk, but heterogeneous in terms of regret aversion. They provide conditions under which the agents who have higher regret aversion simultaneously buy insurance and perform the necessary actions to decrease the probability of incurring losses.

In this paper, we determine conditions under which advantageous selection arises in a model with pure adverse selection (i.e., no moral hazard) and minimal heterogeneity between the agents (i.e., the agents differ only in terms of ex-ante risk). A key aspect of our approach is that we consider loss-averse agents with reference dependent utilities. Furthermore, we show that advantageous selection arises under loss aversion in a wider set of environments than under risk aversion.

A large literature in experimental economics has provided support for the loss aversion framework.[9] The loss aversion framework has been used to explain a variety of empirical puzzles. For example, reference-dependent/loss-averse behavior has been offered as an explanation for the negative elasticity of working hours with respect to wage shown by NYC cab drivers (Crawford and Meng, 2011), for the lower accuracy of putts below par (comparing birdies versus eagles) by professional golf players (Pope and Schweitzer, 2011), and for the intra-personal time inconsistencies made by "Deal or No Deal" contestants (Post et al., 2008). Moreover, loss aversion has been used to explain the observed plainness of contractual arrangements between employers and their employees (Herweg, Müller and Weinschenk, 2010) and why compensation schemes often do not penalize failures (de Meza and Webb, 2007). We complement this literature by suggesting that the loss aversion framework can explain another empirical puzzle: advantageous selection in insurance markets for modest-scale risks.

II. Model

We consider an environment with two agents that are exposed to the risk of incurring (monetary) losses, and a seller that is offering them an insurance to hedge against that risk. The losses that each agent $i \in \{1, 2\}$ is exposed to are represented by a non-negative random variable X_i with cumulative distribution function F_i. Our results hold for both discrete and continuous distributions.

Agents have reference-dependent utilities (which we describe in section III). Each agent is assumed to have the same utility function; however, agents will generally have different reference points $\vec{r}_i$. We assume that each agent is privately informed about his loss distribution and his reference points. We refer to the tuple $(X_i, \vec{r}_i)$ as agent i's *type*. We call *good* (*bad*) the type with the lowest (highest) expected loss $\mathbb{E}X_i$. We designate agent $i = 1$ to be the good type, thus $\mathbb{E}X_1 < \mathbb{E}X_2$.

The timing of the interaction between the seller and the agents is as follows. First, the seller sets a price for the insurance. Then, the agents learn the price and each agent decides whether to buy insurance.

The seller offers insurance for a price p. The insurance is a contract that obligates the seller (the insurer) to fully compensate any buying agent (the insuree) for any loss suffered, irrespectively of the agent's loss distribution. We assume that the seller is risk neutral. This entails that, when agent i buys the insurance product, the seller's profit is $p - X_i$, and thus her expected utility is $p - \mathbb{E}X_i$. When agent i does not buy the insurance product, the seller's utility is zero.

[9]Recent examples include Chua and Camerer (2004); Booij and de Kuilen (2009); Abdelllaoui, Bleichrodt and Paraschiv (2007). For reviews, see DellaVigna (2009); Camerer (2000); Camerer and Loewenstein (2004). We note that some authors (List, 2003; Levitt and List, 2008; Hart, 2005) have criticized the results collected in laboratory experiments arguing that, in some cases, behavior induced by framing effects may have been misinterpreted as supporting evidence for a loss-averse preference specification.

Given the price of the insurance, each agent decides whether to buy. We assume that an agent chooses an action only if it is a *Preferred Personal Equilibrium (PPE)* to do so (Kőszegi and Rabin, 2006); we describe this concept in section III.B. If an agent buys insurance, his consumption is $w - p$, where w denotes the level of initial wealth. If an agent does not buy insurance, his consumption is $w - x$, where x is the realization of the random variable X_i. We assume that the initial wealth w is the same for both agents.

The equilibrium concept that we use to solve the game between the seller and the agents is the *Stackelberg outcome in Preferred Personal Equilibria (SPPE)* (which we define formally in section VI). This is a profile of actions such that: the seller chooses the insurance price that maximizes her profits, and each agent plays the action that is his PPE. This solution concept is a natural extension of subgame-perfect equilibrium to a setting with loss-averse agents: instead of choosing the action that maximizes some utility function, each agent plays his PPE. We restrict our analysis to games where players only use pure strategies, that is, the seller does not offer lotteries over prices and the agents do not randomize between buying and not buying insurance. An equilibrium with *advantageous selection* arises if it is a PPE for the good type to buy insurance, it is a PPE for the bad type to not buy insurance, and the seller maximizes her profit.

III. Reference-Dependent Behavior

A. Reference-Dependent Utility

In a setting with loss aversion, the utility of a given consumption level is affected by how it compares to a *reference point*, with agents exhibiting greater aversion to losses than appreciation for gains. Following previous literature (Kőszegi and Rabin, 2006), we model the agents' loss aversion through a utility function $u(c|r)$, that depends on two consumption levels: c, the actual consumption, and r, the reference point-consumption.

We assume that each agent's utility is a weighted sum of two terms: a consumption term, and a gain-loss term, which depends on the difference between the actual consumption and the reference point. We are interested in attitudes toward small and modest-scale risk (such as \$100 or \$2,000). For such risks, the consumption term can be assumed to be approximately linear.[10] We thus assume that the consumption utility is the identity function.[11] To isolate the consequences of loss aversion, we assume that the gain-loss utility is piecewise linear.[12] More specifically, given a consumption level c and a reference r, agent i's utility is $u(c|r) = c + \eta \cdot \mu(c - r)$. The parameter $\eta > 0$ shows how much the agent weights the gain-loss term compared to the consumption term.

The gain-loss utility μ is given by $\mu(x) = x$ for $x > 0$ and $\mu(x) = \lambda x$ for $x < 0$. The parameter λ is the *loss aversion coefficient* and is assumed to be strictly

[10]See Kahneman and Tversky (1979) and Kőszegi and Rabin (2007).
[11]We note that our main insights also apply to concave consumption utilities.
[12]This is a standard assumption also taken by Kőszegi and Rabin (2006, 2007).

greater than 1, so that μ is steeper for losses than for gains. A larger λ implies that the agent is more loss averse.

Different agents could exhibit different degrees of loss aversion; such heterogeneity can be modeled with different values of η and/or λ. However, in order to isolate the effect of the reference on decisions, we are going to assume homogeneity in loss aversion across agents. In particular, we assume that both agents have exactly the same utility function which we denote with u. When agent i's consumption level is given by a random variable Y_i and the reference point is r_i, agent i's expected utility is $\mathbb{E}[u(Y_i|r_i)] = \mathbb{E}Y_i + \eta \cdot \mathbb{E}[\mu(Y_i - r_i)]$.

B. Agent's Decision Problem

In a loss aversion setting, an agent's utility is a function of both the consumption level and the reference point. Freedom in selecting the reference points may imply indeterminacy in identifying which action is selected: different reference points support different selections. Following Kőszegi and Rabin (2006), we resolve this issue by endogenizing the reference points (each agent's reference point becomes part of his personal equilibrium). This is done by modeling reference points as functions of the actions that the agent expects to take. For example, if agent i expects to take action a, then agent i's reference point becomes r_i^a. In equilibrium, each agent expects to take exactly the action he actually takes. In that way, in equilibrium, for each agent, the reference point and the selected action are consistent.

Let A_i be the set of feasible actions of agent i, and a an element of this set. Let Y_i^a be the (random) consumption associated with action a. That is, the agent knows that if he selects action a, then his future consumption level will be given by Y_i^a and the realization of this random variable will occur in the future. Agent i's reference point associated with action a is denoted by r_i^a. We discuss our assumptions on r_i^a in Section III.C; in general, r_i^a may depend on the distribution Y_i^a and on psychological factors.

DEFINITION 1: *Action $a \in A_i$ is a* personal equilibrium *(PE) for agent i if, for every action $a' \in A_i$, $\mathbb{E}[u(Y_i^a|r_i^a)] \geq \mathbb{E}[u(Y_i^{a'}|r_i^a)]$.*

In words, an action a is agent i's PE if his expected utility is maximized at a when he "expects" to select a, i.e., when his reference point is r_i^a.

There may be multiple PE; in particular, several actions may have the property that if an agent expects to play a given action, that same action is the expected utility maximizer. Since the agent has a single utility function, he can rank the PE outcomes in terms of ex ante expected utility. Then, following Kőszegi and Rabin (2006), we assume that the agent will select the PE that is associated with the highest ex ante expected utility.

DEFINITION 2: *Action $a \in A_i$ is a* preferred personal equilibrium *(PPE) for agent i if (i) it is a PE and (ii) $\mathbb{E}[u(Y_i^a|r_i^a)] \geq \mathbb{E}[u(Y_i^{a'}|r_i^{a'})]$ for every action $a' \in A_i$ that is a PE.*

An alternative equilibrium concept is the choice-acclimating personal equilibrium (CPE) (Kőszegi and Rabin, 2007). An action is a CPE if it maximizes $\mathbb{E}[u(Y_i^a|r_i^a)]$ over all actions, without the requirement of being a PE. A CPE always exists and it is unique (apart from degenerate cases): this may explain the adoption of CPE in the loss aversion literature (e.g., Herweg, Müller and Weinschenk, 2010). According to Kőszegi and Rabin (2007) the applicability of CPE instead of PPE has to do with the timing of expectation formation and decision making.[13] However, in the context of insurance markets, the uncertainty is resolved at an unknown point of time (i.e., the loss can incur at any time) after the decision is made. Under CPE, the action and the reference point are bundled together, and the choice of one implies the choice of the other. This means that CPE may be sub-optimal: an agent who plays a CPE does not consider profitable deviations in which his action and his expectation differ. On the contrary, with PPE, beliefs and actions are considered as independent entities that may mis-match off-equilibrium.

A different factor though plays the most critical role in our choice between equilibrium concepts. We adopt the preferred personal equilibrium (PPE), because CPE may predict implausible behavior in our setting: an agent may buy insurance even if the price is *greater* than the maximum possible loss. In other words, under CPE an agent's willingness to pay for insurance may be unrealistically high. We note that Kőszegi and Rabin (2007) have demonstrated that with CPE people may choose stochastically dominated options and have argued that such counterintuitive behavior may be consistent with some experimental results (Gneezy, List and Wu, 2006) obtained in different contexts than ours; this does not seem plausible in the setting of insurance decisions that we consider in this paper.[14] Nevertheless, we note that the insights of this paper also hold under CPE, and for completeness we consider CPE in Appendix A.

C. Reference Points

A reference point depends on the consumption level induced by the action it corresponds to. If an action brings a deterministic level of consumption, then the corresponding reference point is equal to that consumption level. On the other hand, if an action is associated with a random outcome, a variety of potential reference points arises; the reference point could be the expected consumption level, a convex combination of the values in the support of the probability distribution over consumption levels or a random variable that represents the consumption level.[15]

[13] Kőszegi and Rabin (2007) suggest that whenever the uncertainty associated with an action is resolved long (resp. shortly) after the decision is made, then CPE (resp. PPE) is more appropriate as an equilibrium concept.

[14] Gneezy, List and Wu (2006) identify two characteristics of the experiment design under which a risky prospect to be valued less than its worst possible realization. These characteristics are (1) between-subject tests and (2) having a lottery currency that is different than the pricing currency. Neither of these elements are present in our setting.

[15] See (Kőszegi and Rabin, 2006; Schmidt, Starmer and Sugden, 2008).

In this paper, we focus on general deterministic reference points. This allows us to study the role of reference points on agents' decisions and to consider how agents' decisions change as reference points change. Moreover, we analyze situations where the reference point is given by a specific statistic of the consumption level, such as the expected value or the median. We consider the case of stochastic reference points in Appendix A.

In the specific context of the insurance market we introduced in Section II, the following table summarizes the consumption levels and the reference points that are associated with each action for agent i.

a (action)	Y_i^a (consumption level)	r_i^a (reference point)
buy insurance	$w-p$	$w-p$
not buy insurance	$w-X_i$	$w-k_i$

The reference point associated with buying insurance is equal to $w-p$, reflecting the expense p that an agent has to pay in order to buy insurance. Note that in this case the reference point is the same for both agents.

The reference point associated with not buying insurance is $w-k_i$, where k_i is the *reference loss level* for agent i. The reference loss level k_i is a threshold (expressed in terms of consumption levels) that defines which consumption outcomes are perceived by agent i as losses and which ones as gains. We assume that the value of k_i belongs to the convex hull of the support of X_i in order to rule out situations where the agent perceives all potential outcomes as losses (resp. gains).

What determines the reference loss level? The expected value of the loss distribution is a plausible answer. Other statistics such as the median (de Meza and Webb, 2007) and the mode (Bell, 1985) have also been identified as plausible explanations in the literature.

Moreover, other factors (apart from the distribution of losses) may also affect the reference loss level. According to Bell (1985), "while a mathematician may expect the probabilistic average, an optimist may expect more, a pessimist less." On the other hand, a more responsible agent perceives even a small loss as significant and, as a result, may have a smaller reference loss level. The reference loss level may also depend on the agent's affection (Hsee and Kunreuther, 2000), that is, his attitudes and feelings toward the object that he is considering to insure; and his sense of guilt or shame for any damages to the object.

In general, several factors (e.g., distribution of losses, characteristics of the agent, the relation of the agent with the object) determine which outcomes bring a feeling of loss to the agent and thus the reference loss level. In what follows, we derive conditions for advantageous selection with respect to the reference loss levels k_1 and k_2. We then analyze settings in which the reference loss level k_i represents a specific statistic of the distribution X_i for both agents (and does not depend on any other factors).[16] For instance, k_i could be the expected value,

[16]If $k_i = K(X_i)$ (for some function K), then effectively the reference point is $w-K(X_i)$. Alternatively,

the median or the mode of X_i. In these cases, the only source of heterogeneity across the agents is their random consumption X_i. On the other hand, we also give an example where there is additional heterogeneity across the agents (e.g., with respect to affection).

In Section II we introduced agent i's type as a tuple $(X_i, \vec{r}_i)$, where $\vec{r}_i$ represents the reference points of agent i. Given the complete specification of agent i's utility function, now we can be more precise about the definition of each agent's type in our setting. We consider a model with a different reference point associated with each action, but only the reference loss level associated with the action of not buying insurance may differ across agents and is assumed to be private information. In that sense, agent i's type can be redefined as (X_i, k_i).

IV. Personal Equilibria for Insurance Decisions

In this section, we consider personal equilibria in the context of an insurance market. We derive conditions under which "buying" and "not buying" insurance are PE and PPE for an agent. Given that we focus on a single agent, we drop the index i.

By Definition 1, it is a PE to buy if $p - \mathbb{E}X \leq -\eta \cdot E[\mu(p - X)]$, and it is a PE to not buy if $p - \mathbb{E}X \geq \eta \cdot \mu(k - p) - \eta \cdot \mathbb{E}[\mu(k - X)]$. When both buying and not buying are PE (i.e., both the aforementioned inequalities hold), it is a PPE to buy if $p - \mathbb{E}X \leq -\eta \cdot \mathbb{E}[\mu(k - X)]$, whereas it is a PPE to not buy if $p - \mathbb{E}X \geq -\eta \cdot \mathbb{E}[\mu(k - X)]$ (Definition 2).

We observe that each of these inequalities depends on either $\mathbb{E}[\mu(p - X)]$ or $\mathbb{E}[\mu(k - X)]$. Both of these quantities are the expected value of the gain-loss utility as a function of the difference between some constant (which is either p or k) and the random variable X. The following lemma shows that such an expectation depends on the area below the distribution function of the random variable X.

LEMMA 1: *Let X be a non-negative random variable and let F be its cdf. Then for any $c > 0$, $-\mathbb{E}[\mu(c - X)] = (\lambda - 1)\int_0^c F(x)dx - \lambda(c - \mathbb{E}X)$.*

We now use Lemma 1 to rewrite the conditions for PE. Denote $y^+ \equiv \max(y, 0)$ as the positive part of y.

LEMMA 2: *Define $\psi \equiv \eta(\lambda - 1)/(\eta\lambda + 1)$.*

(i) "Buying" is a PE iff $p - \mathbb{E}X \leq \psi \int_0^p F(x)dx$.

(ii) "Not buying" is a PE iff $\psi\left(\int_0^k F(x)dx - (k - p)^+\right) \leq p - \mathbb{E}X$.

Recall that λ is the loss aversion parameter and η represents the relative weight of the gain-loss term (compared to the consumption term) in an agent's utility.

we could define the reference point to be $K(w - X_i)$. If $K(w - X_i) = w - K(X_i)$ (as is the case for most natural specifications, e.g., $K(X) = \mathbb{E}X$, $K(X) = \text{median}(X)$, $K(X) = \text{mode}(X)$), the two approaches are equivalent.

Thus, the slope of the utility for gains is equal to $\eta + 1$, whereas the slope for losses is $\eta\lambda + 1$. Loss aversion implies that the slope for losses is greater than the slope for gains. The parameter ψ is equal to the ratio of the difference between the two slopes over the slope for losses. Equivalently, ψ is equal to 1 minus the ratio of the slope of gains over the slope at losses. Note that ψ is increasing in both λ and η. We conclude that the parameter ψ is a measure of loss aversion that combines η and λ. Note that $\psi \in (0, 1)$; a value $\psi = 0$ indicates no loss aversion. For the remainder of the paper, we will use ψ — instead of explicitly using λ and η — and refer to ψ as the *degree of loss aversion*.

We next give some general properties of PE and PPE in an insurance market. Our first result shows that a PE always exists, that is, for any instance either "buying" is a PE or "not buying" is a PE or both "buying" and "not buying" are PE.

PROPOSITION 1: *PE and PPE always exist.*

In this paper, we only consider pure strategies. According to Proposition 1, a PE always exists (in pure strategies), that is, either "buying" or "not buying" or both will be PE. We note that in other settings, mixed strategies may be required to guarantee existence of PE (Kőszegi, 2010).

The following lemma shows a connection between PE and PPE depending on whether the price p is greater or smaller than the reference loss level k. These properties are used in the following section.

LEMMA 3:

(i) If $p \geq k$ and "buying" is a PE, then "buying" is a PPE.

(ii) If $p \leq k$ and "not buying" is a PE, then "not buying" is a PPE.

The utility specification of the agents is such that they are at most risk neutral. Given that, it is a PPE to buy insurance if the price is lower than the expected damage; when $p < k$ also holds then "buying" is the unique PE.

V. Willingness to Pay and Reference Point

This section considers an agent's willingness to pay for insurance. As in the previous section, we focus on a single agent and drop the index i.

The following proposition shows that there exists a threshold price such that (i) it is a PPE to buy insurance when the price for insurance is below the threshold, and (ii) it is a PPE to not buy insurance when the price for insurance is above the threshold. Thus, the threshold (which we denote by $\overline{p}$) represents the agent's willingness to pay[17] for insurance (in terms of PPE).[18] The proposition also char-

[17] To be rigorous, if $k > \overline{p}$, then $\overline{p}$ is the *supremum* of the amounts that the agent would be willing to pay for insurance. The willingness to pay is normally defined as the maximum amount that a person is willing to pay; however, in this case the agent is willing to pay any price that is strictly smaller than $\overline{p}$, so the maximum does not exist.

[18] Note that for almost all combinations of prices and reference loss levels, there exists a unique PPE.

acterizes $\overline{p}$ as a function of the distribution of losses F, the reference loss level k and the degree of loss aversion ψ.

PROPOSITION 2: *For given F and ψ, define*

$$\overline{p} = \begin{cases} k^* & \text{if } k \leq k^* \\ \frac{1}{1-\psi}\left(\mathbb{E}X - \psi\left(k - \int_0^k F(x)dx\right)\right) & \text{if } k > k^* \end{cases} \tag{1}$$

where k^* is such that $k^* - \psi \int_0^{k^*} F(x)dx = \mathbb{E}X$. Then,

(i) If $p < \overline{p}$, then "buying" is the unique PPE.

(ii) If $p > \overline{p}$, then "not buying" is the unique PPE.

(ii) Suppose that $p = \overline{p}$. "Buying" is a PPE if and only if $k \leq p$. "Not buying" is a PPE if and only if $k \geq p$.

The parameter k^* is determined solely by the distribution X and the degree of loss aversion ψ, and does not depend on the reference loss level k. If the reference loss level is smaller than k^*, then the willingness to pay is equal to k^*. Therefore, it is possible to know an agent's willingness to pay without knowing the exact value of his reference loss level; in particular, knowing that $k < k^*$ implies that the willingness to pay is equal to k^*. An important property of k^* is that it is lower bounded by the expected loss and upper bounded by the maximum loss. Notice, however, that k^* is equal to the maximum loss only if $\psi = 1$, that is, when the agent is infinitely loss averse. For the remainder of the section we use the following notation: $\overline{p}(\psi, k, X)$ and $k^*(\psi, X)$.

Equation (1) of Proposition 2 can be used to compute the willingness to pay for specific values of the reference loss level k. We next consider two examples, where the reference loss level is equal to (i) the expected value of the loss and (ii) the maximum loss that may occur.

COROLLARY 1:

(i) $\overline{p}(\psi, \mathbb{E}X, X) = k^*(\psi, X)$.

(ii) When $\max X$ is the maximum value in the support of X, $\overline{p}(\psi, \max X, X) = \mathbb{E}X$.

Corollary 1(ii) shows that if the reference loss level k is equal to the maximum loss that may occur, then the agent's willingness to pay is equal to the expected value of losses. In other words, if the agent has the smallest possible reference point, then he is effectively acting as if he were risk neutral. The reason is that in this case the consumption level $w - X$ is always greater than the reference point for not buying insurance.

The only case where the PPE is not unique is if $k = k^* = p$. We use the convention that the agent buys insurance if both "buying" and "not buying" are PPE.

The following proposition provides monotonicity properties of the willingness to pay for insurance with respect to the reference loss level and the distribution of losses.

PROPOSITION 3:

(i) *For fixed ψ and X, $\overline{p}(\psi, k, X)$ is nonincreasing in k. In particular, $\overline{p}(\psi, k, X)$ is constant for $k < k^*(\psi, X)$ and strictly decreasing for $k > k^*(\psi, X)$.*

(ii) *Let X_A and X_B be two non-negative random variables with finite support. If X_B (first order) stochastically dominates X_A, then $\overline{p}(\psi, k, X_B) \geq \overline{p}(\psi, k, X_A)$ for any k, ψ.*

(iii) *For fixed k and X, $\overline{p}(\psi, k, X)$ is increasing in ψ.*

An agent is willing to pay less for insurance when the reference loss level k is larger, or equivalently, when his reference point $w - k$ for not buying insurance is smaller. Indeed, an insurance product that provides a hedge against losses becomes less appealing when the reference point is small, because the cases in which the insurance allows the agent not to fall below his reference point (and suffer the extra disutility connected with the loss feeling) are more limited.

The willingness to pay is also increasing in the distribution of losses. In particular, for a fixed reference loss level, the agent is willing to pay more for insurance as the distribution of losses is more skewed towards higher values. This is an intuitive result in the spirit of adverse selection. It implies that whenever the loss distribution of the bad type stochastically dominates the loss distribution of the good type, a necessary condition for advantageous selection is that the bad type has a higher reference loss level.

In general, the reference loss level may depend on the loss distribution. In particular, we expect the reference loss level to be increasing in the distribution of losses. Then, as described in Proposition 3, a larger distribution of losses has two effects. On one hand, the direct effect of a larger loss distribution is to increase the willingness to pay for insurance, according to part (ii). On the other hand, it implies a larger reference loss level which decreases the willingness to pay, according to part (i). The interaction of these two effects determines whether the willingness to pay increases or decreases.

The monotonicity properties of $\overline{p}(\psi, k, X)$ (given in Proposition 3) and Corollary 1(ii) imply that the agent's willingness to pay is strictly greater than $\mathbb{E}X$ whenever k is strictly smaller than the maximum value in the support of X. Ergo, an agent's willingness to pay for insurance is in the interval $[\mathbb{E}X, k^*(\psi, X)]$. Figure 1 illustrates an agent's willingness to pay for insurance as a function of his reference loss level.

We conclude this section by noting that our results on the willingness to pay of a loss averse agent are natural and intuitive. First, an agent plays a threshold strategy, according to which he buys insurance only below a certain price. Second, the premium that a loss averse agent is willing to pay increases with his loss

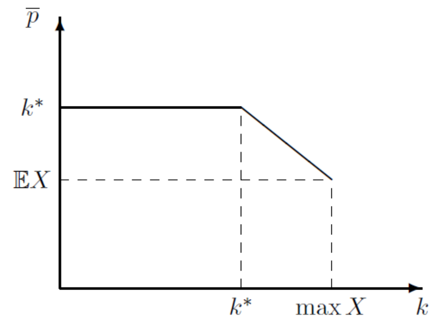

FIGURE 1. WILLINGNESS TO PAY AS A FUNCTION OF THE REFERENCE LOSS LEVEL.

aversion (that is as ψ increases). When the agent is not loss averse (that is, $\psi = 0$), then his willingness to pay is equal to his expected loss. Third, an agent's willingness to pay for insurance is lower bounded by his expected loss and upper bounded by the maximum loss (in contrast to the willingness to pay in terms of CPE which may exceed the maximum loss).

VI. Advantageous Selection in Equilibrium

We start by formally introducing an equilibrium concept for the game between the seller and the agents. We then derive conditions under which advantageous selection arises in equilibrium.

A. Equilibrium Definition

The game we are considering is a two-stage game with three players: the insurance provider moves first and then the agents play in the second stage. The agents do not interact: the action of one agent does not affect the other agent's payoff. Given this structure, we can consider this strategic interaction as a variant of a Stackelberg game where the insurance provider is the leader and both agents are followers.

Let $I = \{s, 1, 2\}$ be the set of players, where s denotes the seller (who is the leader) and $\{1, 2\}$ is the set of followers. We use i to denote a given player and $-i$ to denote all players other than i. Let A_i be the set of available actions of agent i, a_i one element of A_i, and $a = (a_s, a_1, a_2)$ a profile of actions. We define the *PPE best response operator* (in pure strategies) as a mapping of the form $PPE_i : A_{-i} \to A_i$ that, for each player i, given the other players' actions, identifies his PPE action. In the previous sections, given the price p of the insurance, we showed how to identify, for each agent, the action that is PPE. We now propose an equilibrium concept for the game.

DEFINITION 3: *An action profile $a = (a_s, a_1, a_2)$ is a* Stackelberg outcome in Preferred Personal Equilibria *(SPPE) if:*

(i) $a_i = PPE_i(a_s)$ *for* $i \in \{1, 2\}$

(ii) $a_s = PPE_s(PPE_1(a_s), PPE_2(a_s))$

The fact that PPE exists (as shown in Proposition 1) guarantees the existence of SPPE.[19] In our setting, the agents are loss averse, whereas the seller is risk neutral. Notice that risk neutrality may be modeled as a degenerate case of loss aversion, in which $\psi = 0$. Indeed, when $\psi = 0$, the PPE action coincides with the action that maximizes the consumption utility. In that sense, the equilibrium definition above can be further adapted to the specific characteristics of our environment by setting $a_s \in \arg\max_{a'_s} u_s(PPE_1(a'_s), PPE_2(a'_s))$, where u_s denotes the consumption utility of the seller. That is, the seller maximizes his expected profit taking into account that the agents are going to PPE best-respond to him.

B. Conditions for Advantageous Selection

Advantageous selection occurs in SPPE if it is optimal for the seller to charge a price such that agent 1 buys the insurance and agent 2 does not buy.[20] We note that such a price may not always exist; a necessary condition for advantageous selection is that agent 1 is willing to pay more for insurance than agent 2. Moreover, the seller should prefer to sell one insurance contract to the good type instead of two contracts that cover both agents.

The following proposition gives conditions under which advantageous selection occurs in equilibrium.

PROPOSITION 4: *Let* $\overline{p}_i$ *be agent* i*'s willingness to pay. Advantageous selection occurs in SPPE if and only if* $\overline{p}_1 + \mathbb{E}X_2 \geq 2 \cdot \overline{p}_2$*. Moreover, if* $\overline{p}_1 + \mathbb{E}X_2 > 2 \cdot \overline{p}_2$*, this is the unique SPPE.*

If $\overline{p}_1 > \overline{p}_2$, then the seller charges price $\overline{p}_1$ if he intends to only sell to agent 1 and price $\overline{p}_2$ if he intends to sell to both agents. According to Proposition 4, if $\overline{p}_1 + \mathbb{E}X_2 \geq 2 \cdot \overline{p}_2$ or equivalently if $\overline{p}_1 - \overline{p}_2 \geq \overline{p}_2 - \mathbb{E}X_2$, then advantageous selection occurs in equilibrium and the seller charges price $\overline{p}_1$. This is illustrated is Figure 2. The figure on the left shows a case where advantageous selection occurs in the equilibrium. In particular, the distance between $\overline{p}_2$ and $\overline{p}_1$ is longer than the distance between $\mathbb{E}X_2$ and $\overline{p}_2$. In the second figure, advantageous selection does not occur because $\overline{p}_1 < \overline{p}_2$.

From Proposition 2, we know that agent i's willingness to pay is a function of the reference loss level k_i. Thus, given each agents' loss distributions, we can

[19]It is possible to extend this equilibrium concept to multi-stage extensive games where players are active more than once, but that would require to specify how each player's reference points evolve across different stages.

[20]We note that it is possible to also consider advantageous selection in terms of unique PE, and determine conditions under which the seller sets a price such that it is a unique PE for the good type to buy insurance, and it is a unique PE for the bad type not to buy insurance. However, this is a less well defined problem, because a unique PE does not always exist: when both "buying" and "not buying" are PEs, the agent's choice is undetermined. It is then not clear whether the seller would charge a lower price in order to avoid situations of multiplicity of PEs for the agent.

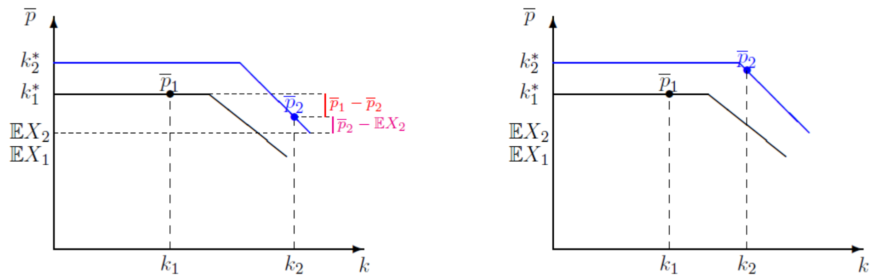

FIGURE 2. THE FIRST (SECOND) GRAPH SHOWS AN EXAMPLE WHERE ADVANTAGEOUS SELECTION OCCURS (DOES NOT OCCUR) IN SPPE.

restate the conditions for advantageous selection in terms of the reference loss levels k_1 and k_2.

Moreover, if the agents' reference loss levels depend on the loss distribution in a specific way, the conditions for advantageous selection in SPPE can be further refined. For example, if $k_i \leq \mathbb{E}X_i$ for $i = 1, 2$, the condition for advantageous selection can be expressed in terms of the parameters k_1^* and k_2^*, each agent's maximum willingness to pay over all potential reference loss levels. In this setting we do not need to know the actual values of k_i to determine if advantageous selection can arise. The setting where the reference loss level is equal to the expected value of the distribution is a special case.

COROLLARY 2: *Let k_i^* be as in Proposition 2. If $k_i \leq \mathbb{E}X_i$ for $i = 1, 2$, then advantageous selection occurs in SPPE if and only if $2 \cdot k_2^* - k_1^* \leq \mathbb{E}X_2$.*

Similar conditions can be determined for other cases. We note, however, that advantageous selection in SPPE is impossible given certain assumptions on the reference loss levels. For instance, if k_1, the reference loss level of the good type, is equal to the maximum value in the support of X_1 (which may be the case if the agent does not suffer any extra disutility in the event of any loss), then, by Corollary 1 (ii), $\overline{p}_1 = \mathbb{E}X_1$. Moreover, $\overline{p}_2 \geq \mathbb{E}X_2 > \mathbb{E}X_1$, which implies that the bad type is willing to pay more than the good type for insurance, so advantageous selection does not occur.

The following proposition identifies a condition under which advantageous selection in SPPE does not occur.

PROPOSITION 5: *Suppose X_1 and X_2 are non-negative random variables with finite support. If X_2 stochastically dominates X_1 and $k_2 \leq \mathbb{E}X_2$, then there exists no SPPE with advantageous selection.*

This implies that when the loss distribution of the bad type stochastically dominates the loss distribution of the good type, then advantageous selection is possible only if the reference loss level of the bad type exceeds the expected value of his loss distribution.

VII. Examples

We now illustrate our results in simple "loss-no loss" settings where each agent either incurs a loss of a given magnitude or no loss. We first assume that the magnitude of the potential loss is the same for both agents. In particular, for $i \in \{1, 2\}$:

$$X_i = \begin{cases} d & \text{with probability } q_i \\ 0 & \text{with probability } 1 - q_i \end{cases} \tag{2}$$

We assume that $q_1 < q_2$ so that agent 1 is the good type. [21] The following proposition provides the necessary and sufficient conditions for advantageous selection in SPPE in this setting.

PROPOSITION 6: *For $i = 1, 2$, suppose that X_i is given by (2) and let $k_i^* = \frac{q_i d}{1-\psi(1-q_i)}$. An equilibrium with advantageous selection in SPPE exists if and only if $k_2 \geq k_2^*$ and either of the following pairs of conditions holds:*

(i) $k_1 < k_1^*$ *and* $k_1^*(1-\psi) \geq q_2\left((1+\psi)\,d - 2\psi k_2\right)$

(ii) $k_1 \geq k_1^*$ *and* $q_1(d - \psi k_1) \geq q_2((1+\psi)d - 2\psi k_2)$

The first set of conditions corresponds to the case that $\overline{p}_1 > k_1$ and $\overline{p}_2 < k_2$, while the second to the case that $\overline{p}_i < k_i$ for both agents. In both cases, k_2 is greater than the willingness to pay of agent 2; this is a necessary condition for advantageous selection in this setting (and more generally, whenever X_2 stochastically dominates X_1).

We next illustrate that SPPE with advantageous selection may arise in this simple "loss-no loss" setting when the reference loss level is determined by the median or the mode of each agent's distribution over losses.

EXAMPLE 1: *Suppose that X_i is given by (2), $k_1 = 0$ and $k_2 = d$. These values for the reference loss levels arise for instance when k_i is equal to the median or the mode of X_i and $q_1 < 0.5 \leq q_2$. Then, we trivially have that $k_1 < k_1^*$ and $k_2 > k_2^*$, and by Proposition 6 (i) advantageous selection occurs in SPPE if and only if $\frac{q_1}{1-\psi(1-q_1)} > q_2 > q_1$. This implies that for every value of q_1, we can find a range of values for q_2 for which advantageous selection occurs in SPPE (assuming that $k_1 = 0$ and $k_2 = d$ are fixed).*

When X_i is given by (2), the loss distribution of agent 2 stochastically dominates the loss distribution of agent 1. Proposition 5 then implies that advantageous selection in SPPE is not possible when the reference loss level of each agent is given by the expected value of his loss distribution. The following example shows that when X_i is given by (2), advantageous selection in SPPE may occur if —

[21] We note that this distribution has been often considered in the literature (e.g., Rothschild and Stiglitz, 1976).

in addition to the expected loss — the reference loss level also depends on other effects, such as affection or responsibility. For instance, a more responsible agent perceives even a small loss as significant and, as a result, has a smaller reference loss level. Thus, a responsible agent with a small expected loss may be willing to pay more for insurance than a less responsible agent with a larger expected loss.

EXAMPLE 2: *Suppose that X_i is given by (2), $k_1 = \mathbb{E}X_1$ and $k_2 = \mathbb{E}X_2 + a$, where $a > 0$. This specification is reasonable if agent 2 is less responsible or has less affection for the insurable item (Hsee and Kunreuther, 2000); the parameter a represents how much less responsible agent 2 is or how much less affection agent 2 has than agent 1 respectively. Proposition 6 implies that advantageous selection occurs in SPPE if*

$$a \geq d \cdot \left(\frac{1+\psi}{2\psi} - q_2 - \frac{(1-\psi)q_1}{(1-\psi(1-q_1))2q_2\psi} \right).$$

For instance, this condition holds if $\psi = \frac{1}{3}$, $q_1 = 0.26$, $q_2 = 0.3$, $d = 10$ and $a = 5.5$.

We next provide an example demonstrating that advantageous selection may arise in SPPE when $k_i = \mathbb{E}X_i$ for a generalization of (2) where the two agents occur losses of different magnitudes.

EXAMPLE 3: *Suppose that, for each $i \in \{1, 2\}$, $k_i = \mathbb{E}X_i$ and*

$$X_i = \begin{cases} d_i & \text{with probability } q_i \\ 0 & \text{with probability } 1 - q_i \end{cases} \tag{3}$$

The condition $q_1 d_1 < q_2 d_2$ guarantees that agent 1 is the good type. Advantageous selection occurs in SPPE if $\frac{q_1 d_1}{1-\psi(1-q_1)} > q_2 d_2 \frac{1+\psi(1-q_2)}{1-\psi(1-q_2)}$, a condition that is satisfied for some values of the parameters (e.g., $q_1 = 0.1$, $q_2 = 0.5$, $d_1 = 40$, $d_2 = 10$, and $\psi = 2/3$).

We note that in this case $q_2 > q_1$ and $d_2 < d_1$ are necessary condition for advantageous selection in SPPE. This implies that the loss distribution of the good type needs to be more spread than the loss distribution of the bad type. In light of Proposition 5 this is not surprising, because $q_2 > q_1$ and $d_2 > d_1$ would imply that X_2 stochastically dominates X_1 so advantageous selection in SPPE would not be possible.

VIII. Risk Aversion

Previous literature has identified conditions for advantageous selection in settings where agents are risk averse. In these models the good type is the one with the highest degree of risk aversion. The negative correlation between expected loss and degree of risk aversion may be a natural phenomenon in models with moral hazard. However, this is not necessarily the case in settings of pure information asymmetry, where any combination of risk aversion degree and loss distribution is plausible.

As in the case of loss aversion, it is possible for advantageous selection to arise in a setting with risk averse agents who have the same utility function (and thus the same degree of risk aversion). In general, however, calibration studies[22] have shown that implausibly high degrees of risk aversion are required to justify the purchase of insurance against small-scale risks. Moreover, we show that for certain families of loss distributions for which advantageous selection can occur with loss aversion, advantageous selection is impossible (when the agents have the same degree of risk aversion.)

Suppose that both agents have the same concave utility function u (at the risk of overloading notation). Then, agent i is willing to buy insurance if and only if $\mathbb{E}[u(w - X_i)] \leq u(w - p)$. Thus, agent i's willingness to pay for insurance is $\tilde{p}_i \equiv w - u^{-1}(\mathbb{E}(u(w - X_i)))$. Similarly to Proposition 4, advantageous selection occurs in equilibrium if and only if $\tilde{p}_1 + \mathbb{E}X_2 \geq 2\tilde{p}_2$. Moreover, if $\tilde{p}_1 + \mathbb{E}X_2 > 2\tilde{p}_2$, this is the unique equilibrium.

EXAMPLE 4: *Suppose that X_i is given by (2) and $q_1 < q_2$. Proposition 6 identifies conditions under which advantageous selection occurs with these loss distribution in the case of loss aversion. In contrast to the case of loss aversion, with risk aversion advantageous selection cannot arise in this setting. In particular, a necessary condition for advantageous selection is that $\tilde{p}_1 > \tilde{p}_2$, which can only hold if $\mathbb{E}[(u(w - X_1))] < \mathbb{E}[(u(w - X_2))]$. Since X_i is given by (2), this condition can be rewritten as*

$$(1 - q_1)\, u\,(w) + q_1 u\,(w - d) < (1 - q_2)\, u\,(w) + q_2 u\,(w - d)\,.$$

However, this cannot hold since $q_1 < q_2$ and u is increasing.

Example 4 demonstrates that in order to have an equilibrium with advantageous selection when the loss distributions are given by (2), we need the agents to have different utilities. A necessary condition is that the good type exhibits a higher degree of risk aversion.[23]

In conclusion, besides calibration issues, for advantageous selection to arise, the agents need to show different degrees of risk aversion in a very specific manner, or, alternatively, the type space needs to be sufficiently complex.

[22] See Kőszegi and Rabin (2007)

[23] In particular, the following conditions are necessary for advantageous selection:

$$(1 - q_1)\, u_1\,(w) + q_1 u_1\,(w - d) \leq u_1\,(w - p)\,;$$

$$u_2\,(w - p) \leq (1 - q_2)\, u_2\,(w) + q_2 u_2\,(w - d)\,.$$

Since $q_1 < q_2$ and u_2 is increasing, the latter condition implies that

$$u_2\,(w - p) < (1 - q_1)\, u_2\,(w) + q_1 u_2\,(w - d)\,.$$

Thus, when deciding between a deterministic consumption level $w - p$ and a lottery which gives $w - d$ with probability q_1 and w with probability $1 - q_1$, agent 1 prefers the safe option and agent 2 prefers the lottery. Thus, agent 1 is more risk averse than agent 2 when choosing between $w - p$ and $w - X_1$.

IX. Discussion

Empirical studies have shown cases of advantageous selection in the market of insurance, as ex-post risk and coverage are negatively correlated in several markets. We offer a novel explanation for this phenomenon that relies on the concept of loss aversion and the role of reference points. We show that advantageous selection may be a market equilibrium outcome if agents are loss averse and take into account their loss distributions when they form their reference points. We determine necessary and sufficient conditions under which the seller is better off setting an insurance price that is appealing only to the good type.

Even though throughout the paper we have considered a monopolist insurance provider, our results can be extended to a hybrid setting with competition and insurance "loads" or administrative costs of providing insurance (de Meza and Webb, 2001; Einav and Finkelstein, 2011). In the case of insurance loads, the expected cost for providing insurance to an agent is equal to the sum of the agent's expected loss and the insurance load. If the good type is willing to pay more than the bad type for insurance and the insurance load is such that the total cost of insuring the good type exceeds the willingness to pay of the bad type, then advantageous selection occurs in equilibrium.

A desirable extension is one in which the set of insurance contracts considered includes partial coverage insurance contracts. In such an environment, advantageous selection occurs if the bad type buys a lower degree of coverage than the good type. Assuming that the option of not buying insurance is always available to each agent, then the new model entails the presence of at least three alternatives in each agent's consideration set (no insurance, high coverage insurance, low coverage insurance). It turns out that deriving the Preferred Personal Equilibrium choice in the presence of more than two options may be very complicated. Still, under certain conditions, advantageous selection arises as the equilibrium outcome.

The role of reference points is a key theme in this paper. The idea that reference-dependent utilities allow to capture well how people actually take decisions in several situations is gaining credit over time. Therefore, understanding which factors contribute to determine the reference points is an open issue that challenges different disciplines, such as behavioral economics, psychology and sociology. We have discussed examples where, in deciding whether to insure a given product, the reference point is a function of the potential losses the product may suffer, the level of affection of the owner for the product, the sense of guilt and responsibility of the owner in case a damage occurs to the product. It would be beneficial to understand better which other factors come into play, and which of them are more important.

In this paper, we have analyzed the decision of buying insurance in an environment in which the risk is pre-assigned to the agents. This may be the case because the risk is agent-specific, or because the risk is product-specific and the agent is assumed to already own the product exposed to the risk. A future direction for investigation may be to endogenize the risk distribution across the agents. In this case, we could assume that the risk is product specific and the agents take two decisions: which good to buy and whether to purchase insurance. Often, as in the case of extended warranties for appliances, the insurance product is an add-on: it does not influence the purchase decision over the base good. In that case, it makes sense to treat the insurance decision independently. However, there are markets in which the value of the base-product is very much affected by the level of maintenance and service (insurance) that can be attached to it. This is the case, for example, in the market of hardware for business customers.

Finally, more empirical work is needed to identify the extent of the advantageous selection phenomenon and to test which theory better explains how it works. This is especially true with respect to markets of insurance products covering small scale risks (e.g., extended warranties for appliances), where consumers' purchasing behavior does not seem to be explainable in terms of standard risk aversion.

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MATHEMATICAL APPENDIX FOR ONLINE PUBLICATION

A. Stochastic Reference Points

A reference point depends on the consumption level induced by the action it corresponds to. If an action brings a deterministic level of consumption, then the corresponding reference point is equal to that consumption level. On the other hand, if an action is associated with a random outcome, a variety of potential reference points arises. In our setting, this is relevant for the action of not buying insurance. Throughout the paper, we considered a general deterministic reference point for the action of not buying insurance.

In this appendix, we consider the reference point suggested by Kőszegi and Rabin (2006). If the consumption level associated with an action is given by a random variable Y, then the reference point is given by a random variable with the same distribution. We refer to this as the KR reference point. In this appendix, we study how the adoption of KR reference points affects which equilibria arise. We consider two equilibrium concepts: Preferred Personal Equilibrium (PPE) and Choice-acclimating Personal Equilibrium (CPE).

With KR reference points, we cannot find a general closed-form solution for the willingness to pay in terms of PPE. This prevents us from fully characterizing the set of environments in which advantageous selection occurs in SPPE. Nevertheless, we derive some results for specific environments. We next show that advantageous selection does not occur in SPPE if X_i is given by (2).

PROPOSITION 7: *Assume KR reference points. If X_i is given by* (2)*, there exists no SPPE with advantageous selection.*

PROOF:

We first show that if "buying" is a PE then it is a PPE. "Buying" is a PE for agent i if $p - \mathbb{E}X_i \leq -\eta\mathbb{E}[\mu(p - X_i)]$. (Note that this condition is the same as in the case of deterministic reference points.) Since X_i is given by (2), "buying" is a PE for agent i if

$$p \leq dq_i \frac{\eta\lambda + 1}{1 + \eta + \eta(\lambda - 1)q_i}. \tag{A1}$$

If both "buying" and "not buying" are PE, then "buying" is PPE if $p - \mathbb{E}X < -\mathbb{E}[\mu(X - X')]$, where the random variable X' has the same distribution as X. From (2),

$$p \leq q_i d[1 + \eta(\lambda - 1)(1 - q_i)]. \tag{A2}$$

Straightforward calculations show that (A1) implies (A2). Thus, if "buying" is a PE then it is a PPE.

We now observe that the right hand side of (A1) is increasing in q_i. This implies that if $q_1 < q_2$ and "buying" is a PE for agent 1, then "buying" is a PE for agent 2. Since we have shown that "buying" is a PPE if and only if it is a PE in this case, we conclude that if $q_1 < q_2$ and "buying" is a PPE for agent 1, then "buying" is a PPE for agent 2. Thus, there does not exist an SPPE with advantageous selection. ∎

In environments characterized by a distribution of potential losses other than (2), SPPE advantageous selection is possible with KR reference points. We provide here an example.

EXAMPLE 5: *Assume that* X_i *is given by* (3), $d_1 = 100$, $q_1 = 0.14$ *and* $d_2 = 30$, $q_2 = 0.5$. *The good type has an expected damage of* 14 *and the bad type has an expected damage of* 15. *Assume that* $\lambda = 4$ *and* $\eta = 1$. *The willingness to pay for insurance is* $\overline{p}_2 \approx 21.4$ *for the bad type and* $\overline{p}_1 \approx 28.9$ *for the good type. Ergo, it is optimal for the seller to charge* $\overline{p}_1$ *and earn a profit of* $\pi = 28.9 - 14 = 14.9$ *(rather than charging* $\overline{p}_2$ *and earning* $\pi = 42.8 - 14 - 15 = 13.8$*).*

The concept of *choice-acclimating personal equilibrium* (CPE) was introduced by Kőszegi and Rabin (2007). In this appendix we demonstrate its shortcoming (which has been discussed in Section III.B). The major difference between CPE and PPE is the timing of expectation formation and actual decision making. An action is a CPE if it maximizes the ex ante expected utility among all possible actions. This is formalized in the following definition.

DEFINITION 4: *An action* $a \in A_i$ *is a* choice-acclimating personal equilibrium *(CPE) for agent* i *if* $\mathbb{E}[u_i(Y_i^a|r_i^a)] \geq \mathbb{E}[u_i(Y_i^{a'}|r_i^{a'})]$ *for every action* $a' \in A_i$.

(Recall that an action is a PPE if it is a PE and maximizes the ex ante expected utility among the set of PE.)

Let X' be a random variable that has the same distribution as X. Applying Definition 4, it is a CPE to buy if $p - \mathbb{E}X_i \leq -\eta \cdot \mathbb{E}[\mu(X_i - X_i')]$, whereas it is a CPE to not buy if $p - \mathbb{E}X_i \geq -\eta \cdot \mathbb{E}[\mu(X_i - X_i')]$. We conclude that agent i's willingness to pay for insurance (in terms of CPE) is

$$\check{p}_i \equiv \mathbb{E}X_i - \eta \cdot \mathbb{E}[\mu(X_i - X_i')].$$

We note that the willingness to pay in terms of CPE is related to the concept of the average self-distance introduced by Kőszegi and Rabin (2007). In particular, the average self-distance of a lottery F is defined (in the appendix of that paper) to be $S(F) = \int\int |x - y| dF(x) dF(y)$. Straightforward calculations show that $\mathbb{E}[\mu(X_i - X_i')] = -\frac{\lambda-1}{2} S(F_i)$. Thus, agent i's willingness to pay for insurance in terms of CPE is equal to $\mathbb{E}X_i + \eta \frac{\lambda-1}{2} S(F_i)$.

Similarly to Definition 3, we can define Stackelberg outcome in Choice-acclimating Personal Equilibria (SCPE): the seller maximizes his expected payoff taking into account that the agents are going to CPE best-respond to him. Advantageous selection in SCPE arises if $\check{p}_1 - \mathbb{E}X_1 > 2\check{p}_2 - \mathbb{E}X_1 - \mathbb{E}X_2$.

The following example shows that with CPE an agent's willingness to pay for insurance may exceed the maximum possible loss. As we have discussed in Section

III.B, this implausible behavior led us to study PPE (instead of CPE) throughout the paper. Nevertheless, the following example also shows that advantageous selection is also possible with CPE.

EXAMPLE 6: *Suppose that X_i is given by (2). Then agent i's CPE willingness to pay for insurance is $\check{p}_i = q_i d(1+\eta(\lambda-1)(1-q_i))$. Let $h(q) = qd(1+\eta(\lambda-1)(1-q))$, so that (fixing d, λ and η) the CPE willingness to pay of agent i is given by $h(q_i)$ (i.e., $\check{p}_i = h(q_i)$). As we would expect, for deterministic outcomes (that is, for $q=0$ and $q=1$) the willingness to pay for insurance is equal to the deterministic loss.*

However, implausible behavior may arise for stochastic outcomes: the willingness to pay may be greater than the maximum possible loss (that is, $\check{p}_i > d$).[24] *In particular, $h(q) > d$ for any $q \in [(1+\eta(\lambda-1))/(2\eta(\lambda-1)), 1)$. This is related to Proposition 7 of Kőszegi and Rabin (2007) on CPE, which shows that for sufficiently large η the agent selects the best deterministic option even if it is stochastically dominated by some stochastic option.*

We now assume that $\eta(\lambda-1) > 1$ and $(1+\eta(\lambda-1))/(2\eta(\lambda-1)) < q_1 < q_2$. Since h is decreasing for $q \in [(1+\eta(\lambda-1))/(2\eta(\lambda-1)), 1]$, we have that agent 1 is willing to pay more than agent 2 for insurance — even though agent 2 has a greater probability to incur a loss. Advantageous selection occurs in SCPE if $q_1(1+\eta(\lambda-1)(1-q_1)) > q_2(1+2\eta(\lambda-1)(1-q_2))$, which is the case if $\eta(\lambda-1)$ is sufficiently large.

B. Proofs of Propositions and Lemmas

PROOF OF LEMMA 1:

First suppose that X is a continuous distribution with density f. Then,

$$\begin{aligned}
-\mathbb{E}[\mu(c-X)] &= \int_0^\infty (x-c)f(x)dx + (\lambda-1)\int_c^\infty (x-c)f(x)dx \\
&= \lambda\int_0^\infty (x-c)f(x)dx - (\lambda-1)\int_0^c (x-c)f(x)dx \\
&= \lambda(\mathbb{E}X - c) + (\lambda-1)\int_0^c (c-x)F'(x)dx \\
&= -\lambda(c-\mathbb{E}X) + (\lambda-1)\int_0^c F(x)dx
\end{aligned}$$

The last equality follows by integration by parts.

The proof for a discrete distribution is very similar. The first step is to show that $-\mathbb{E}[\mu(c-X)] = -\lambda(c-\mathbb{E}X) + (\lambda-1)\sum_{x_i<c}(c-x)\mathbb{P}[X=x_i]$, and then observe that $\sum_{x_i<c}(c-x_i)\mathbb{P}[X=x_i] = \int_0^c F(x)dx$, because this is the area under the cdf F from 0 to c.

PROOF OF LEMMA 2:

[24] We note that this issue also arises with deterministic reference points.

(i) follows from the fact that buying is a PE if $p - \mathbb{E}X \leq -\eta \cdot E[\mu(p-X)]$ and Lemma 1.

We now show (ii). Applying Lemma 1, we see that "not buying" is a PE if and only if

$$p - \mathbb{E}X + \eta\lambda(k - \mathbb{E}X) - \eta \cdot \mu(k-p) \geq \eta(\lambda - 1)\int_0^k F(x)dx. \tag{B1}$$

We now consider two cases. First, if $k \leq p$, then $\mu(k-p) = -\lambda(p-k)$ and the left hand side of (B1) is equal to $(1+\eta\lambda)(p - \mathbb{E}X)$, so (B1) can be rewritten as

$$p - \mathbb{E}X \geq \psi \int_0^k F(x)dx. \tag{B2}$$

where $\psi \equiv \eta\frac{\lambda-1}{\eta\lambda+1}$.

The second case is $k \geq p$. Then (B1) can be rewritten as

$$p - \mathbb{E}X \geq \psi\left(\int_0^k F(x)dx - (k-p)\right). \tag{B3}$$

We get the result by combining (B2) and (B3).

PROOF OF PROPOSITION 1:

By Lemma 2, neither "buying" nor "not buying" is a PE if and only if

$$\psi\int_0^p F(x)dx < p - \mathbb{E}X < \psi\left(\int_0^k F(x)dx - (k-p)^+\right).$$

However this cannot hold because

$$\int_0^k F(x)dx - (k-p)^+ \leq \int_0^p F(x)dx.$$

PROOF OF LEMMA 3:

(i) If $p \geq k$, then $-\mathbb{E}[\mu(p-X)] \leq -\mathbb{E}[\mu(k-X)]$. Thus, $p - \mathbb{E}X \leq -\eta \cdot E[\mu(p-X)]$ implies $p - \mathbb{E}X \leq -\eta \cdot \mathbb{E}[\mu(k-X)]$. That is, if "buying" is a PE, then "buying" is a PPE.

(ii) If $p \leq k$, then $\mu(k-p) \geq 0$. Thus, $p - \mathbb{E}X \geq \eta \cdot \mu(k-p) - \eta \cdot \mathbb{E}[\mu(k-X)]$ implies $p - \mathbb{E}X \geq -\eta \cdot \mathbb{E}[\mu(k-X)]$. That is, if "not buying" is a PE, then "not buying" is a PPE.

PROOF OF PROPOSITION 2:

Let

$$g(k,p) \equiv p - \psi\left(\int_0^{\max(p,k)} F(x)dx - (k-p)^+\right).$$

We first show that if $\overline{p}$ satisfies $g(k, \overline{p}) = \mathbb{E}X$, then (i), (ii) and (iii) hold. We consider three different cases for the relation between the reference loss level k and the price p.

- First suppose that $k < p$. Then "buying" is a PPE if and only if it is a PE (by Lemma 3). On the other hand, "not buying" is a PPE if and only if "buying" is not a PE (because by Proposition 1 a PE always exists). We apply Lemma 2 and find that "buying" is a PPE if and only if

$$p - \mathbb{E}X \leq \psi \int_0^p F(x)dx,$$

  and "not buying" is a PPE if and only if

$$p - \mathbb{E}X > \psi \int_0^p F(x)dx.$$

- Now suppose that $k > p$. Then "not buying" is a PPE if it is a PE and "buying" is a PPE if "not buying" is not a PE (these follow from Proposition 1 and Lemma 3). We conclude that "buying" is a PPE if and only if

$$p - \mathbb{E}X < \psi \left(\int_0^k F(x)dx - (k - p) \right),$$

  and "not buying" is a PPE if and only if

$$p - \mathbb{E}X \geq \psi \left(\int_0^k F(x)dx - (k - p) \right).$$

- We finally consider the case that $k = p$. Similar arguments as in the previous cases show that "buying" is a PPE if and only if

$$p - \mathbb{E}X \leq \psi \left(\int_0^k F(x)dx - (k - p) \right),$$

  and "not buying" is a PPE if and only if

$$p - \mathbb{E}X \geq \psi \left(\int_0^k F(x)dx - (k - p) \right).$$

  Note that the only reason we consider this as a special case (instead of including it in one of previous cases) is that both inequalities are weak (whereas in the previous cases one inequality is weak and the other is strict).

It follows from the analysis above that (1) if $g(k, p) < \mathbb{E}X$ then "buying" is the unique PPE, and (2) if $g(k, p) > \mathbb{E}X$ then "not buying" is the unique PPE. We now observe that g is strictly increasing in p and conclude that if $g(k, \overline{p}) = \mathbb{E}X$

then (i), (ii) and (iii) in the statement of this proposition hold.

We observe that $g(k,0) < \mathbb{E}X$ and $\lim_{p\to\infty} g(k,p) = \infty$. Also, g is continuous and strictly increasing in p, which implies that $g(k,p) = \mathbb{E}X$ has a unique solution. Finally, (using the fact that $\max(\overline{p},k) = k^*$ if $k < k^*$) simple calculations show that $g(k,\overline{p}) = \mathbb{E}X$ when $\overline{p}$ is given by (1). This concludes the proof.

PROOF OF COROLLARY 1:

(i) Straightforward calculations show that $\mathbb{E}X < k^*(\psi,X)$. Thus, by Proposition 2, $\overline{p}(\psi,\mathbb{E}X,X) = k^*(\psi,X)$.

To show (ii), we first observe that $\int_0^{\max X} F(x)dx = \max X - \mathbb{E}X$. This implies that the $k^*(\psi,X) < \max X$. Then, by Proposition 2, we have that

$$\overline{p}(\psi,\max X,X) = \frac{1}{1-\psi}\left(\mathbb{E}X - \psi\left(m - \int_0^m F(x)dx\right)\right) = \mathbb{E}X$$

PROOF OF PROPOSITION 3:

(i) and (iii) are a direct consequences of Proposition 2.

To show (ii), let F_A and F_B be the distribution functions of X_A and X_B respectively. Since X_B stochastically dominates X_A, we have that $F_B(x) \leq F_A(x)$ for all x. Our proof is based on the following inequality:

$$\text{(B4)} \qquad \mathbb{E}X_B + \psi\int_0^k F_B(x)dx \geq \mathbb{E}X_A + \psi\int_0^k F_A(x)dx.$$

To see why this holds, observe that $\mathbb{E}X_B = c - \int_0^c F_B(x)dx$ for any constant c that is greater than the maximum element in the support of X_B. Thus,

$$\mathbb{E}X_B + \psi\int_0^k F_B(x)dx = c - (1-\psi)\int_0^k F_B(x)dx - \int_k^c F_B(x)dx$$

for sufficiently large c, which together with the fact that X_B stochastically dominates X_A implies (B4).

Proposition 2 and (B4) imply that if $k \geq \max(k^*(\psi,X_A),k^*(\psi,X_B))$ then $\overline{p}(\psi,k,X_B) \geq \overline{p}(\psi,k,X_A)$. To conclude the proof it suffices to show that $k^*(\psi,X_B) \geq k^*(\psi,X_A)$. Let $k_A = k^*(\psi,X_A)$ and $k_B = k^*(\psi,X_B)$. For the sake of contradiction, suppose that $k_A > k_B$. Then,

$$\begin{aligned}
\mathbb{E}X_B - \mathbb{E}X_A &= \left(k_B - \psi\int_0^{k_B} F_B(x)dx\right) - \left(k_A - \psi\int_0^{k_A} F_A(x)dx\right)\\
&< \left(k_A - \psi\int_0^{k_A} F_B(x)dx\right) - \left(k_A - \psi\int_0^{k_A} F_A(x)dx\right)\\
&= \psi\int_0^{k_A}(F_A(x) - F_B(x))dx\\
&\leq \mathbb{E}X_B - \mathbb{E}X_A,
\end{aligned}$$

which is a contradiction. We note that the first inequality follows from the assumption that $k_A > k_B$ and the second from (B4).

PROOF OF PROPOSITION 4:

From Proposition 2, we know that agent i buys insurance if $p < \overline{p}_i$ and does not buy insurance if $p > \overline{p}_i$. If $\overline{p}_1 \leq \overline{p}_2$, then there does not exist any price at which agent 1 buys and agent 2 does not buy, so advantageous selection cannot occur. We thus assume that $\overline{p}_1 > \overline{p}_2$ for the remainder of the proof.

If $\overline{p}_1 > \overline{p}_2$, then the seller has the following options:

- Charge price $\overline{p}_2$, so that both agents buy the insurance product. The seller's profit then is $2 \cdot \overline{p}_2 - \mathbb{E}X_1 - \mathbb{E}X_2$.
- Charge price $\overline{p}_1$, so that only agent 1 buys the insurance product. The seller's profit then is $\overline{p}_1 - \mathbb{E}X_1$.

An SPPE with advantageous selection arises if the seller's profit from selling only to the good type is at least as large as the profit from selling to both agents, which is the case if $\overline{p}_1 + \mathbb{E}X_2 \geq 2 \cdot \overline{p}_2$.

On the other hand, if $\overline{p}_1 + \mathbb{E}X_2 > 2 \cdot \overline{p}_2$, then the seller strictly prefers to charge price $\overline{p}_1$ and only sell to agent 1. As a result, a unique SPPE exists in this case and advantageous selection occurs at the equilibrium. This concludes the proof.

PROOF OF COROLLARY 2:

We observe that $k_i^* > \mathbb{E}X_i$ for any random variable X_i. Thus, by Proposition 2, if $k_i \leq \mathbb{E}X_i$, then $\overline{p}_i = k_i^*$ and the result follows.

PROOF OF PROPOSITION 5:

By Proposition 3(ii) we have $k_2^* \geq k_1^*$. Given that $k_2 \leq \mathbb{E}X_2$, by Proposition 2, $\overline{p}_2 = k_2^*$. On the other hand, by Proposition 2 and 3(i), $\overline{p}_1 \leq k_1^*$. Ergo $\overline{p}_2 = k_2^* \geq k_1^* \geq \overline{p}_1$. This implies that advantageous selection cannot arise, given that for that to occur we need $\overline{p}_1 - \overline{p}_2 \geq \overline{p}_2 - \mathbb{E}X_2 > 0$.

PROOF OF PROPOSITION 6:

We apply Proposition 2 and find that $k_i^* = \frac{q_i d}{1-\psi(1-q_i)}$. Then, the willingness to pay is given by

$$\overline{p}_i = \begin{cases} k_i^* & \text{if } k_i < k_i^* \\ q_i \frac{d-\psi k_i}{1-\psi} & \text{if } k_i \geq k_i^* \end{cases}$$

We observe that $q_1 < q_2$ implies that $k_1^* < k_2^*$, which means that advantageous selection cannot occur if $k_2 < k_2^*$. Therefore a necessary condition for advantageous selection is $k_2 \geq k_2^*$. This implies that $\overline{p}_2 = q_2 \frac{d-\psi k_2}{1-\psi}$, and $2\overline{p}_2 - \mathbb{E}X_2 = 2q_2 \frac{d-\psi k_2}{1-\psi} - q_2 d = q_2 \frac{(1+\psi)d-2\psi k_2}{1-\psi}$. By Proposition 4 we have advantageous selection in SPPE if $\overline{p}_1 \geq 2\overline{p}_2 - \mathbb{E}X_2$. We consider the following cases:

(i) If $k_1 < k_1^*$ then $\overline{p}_1 = k_1^*$ and advantageous selection occurs in SPPE if $k_1^* \geq q_2 \frac{(1+\psi)d-2\psi k_2}{1-\psi}$.

(ii) If $k_1 \geq k_1^*$ then $\overline{p}_1 = q_1 \frac{d-\psi k_i}{1-\psi}$ and advantageous selection occurs in SPPE if $q_1 \frac{d-\psi k_i}{1-\psi} \geq q_2 \frac{(1+\psi)d-2\psi k_2}{1-\psi}$.