Appendix IV: Incorporating SRT (Supplier Response Time) into the Safety Stock Calculations

A weekly review period is assumed.

Let: X = actual amount of demand for an arbitrary part in $L+1$ weeks.

$$S = \text{Order-up-to level} = \sum_{i=1}^{L+1} P_i + Z_{1-\alpha}\sigma_X.$$

X is assumed to be normally distributed with mean $(L+1)\mu_D$ and variance $\sigma_D^2(L+1) + \sigma_L^2\mu_D^2$.

Then $\text{Prob}(X \le S) = 1 - \alpha$, so $1 - \alpha$ is the service level.

One-Week SRT

The probability that some demand is actually filled the week following its arrival is the probability that the order-up-to level over $L+1$ weeks covers demand incurred over just L weeks.

Let X^* be the amount of demand in L weeks. X^* is normally distributed with mean $L\mu_D$ and variance $\sigma_D^2 L + \sigma_L^2\mu_D^2$.

If SS_1 denotes the appropriate safety stock for a one-week SRT, the corresponding order-up-to level for a one-week SRT goal is $S_1 = \sum_{i=1}^{L+1} P_i + SS_1$. However,

$$\text{Prob}\left(X^* \le S_1\right) = \text{Prob}\left(Z \le \frac{S_1 - L\mu_D}{\sqrt{\sigma_D^2 L + \sigma_L^2\mu_D^2}}\right) = 1 - \alpha.$$

This implies that

$$Z_{1-\alpha} = \frac{S_1 - L\mu_D}{\sqrt{\sigma_D^2 L + \sigma_L^2\mu_D^2}}$$

$$S_1 = Z_{1-\alpha}\sqrt{\sigma_D^2 L + \sigma_L^2\mu_D^2} + L\mu_D.$$

The order-up-to-level will still be calculated by our in-house procurement system, POPLAN, as $S_1 = \sum_{i=1}^{L+1} P_i + SS_1$, so we now have two expressions for S_1. Assuming that $\mu_D = \bar{P}_{L+1}$,

$$S_1 = (L+1)\mu_D + SS_1 = Z_{1-\alpha}\sqrt{\sigma_D^2 L + \sigma_L^2\mu_D^2} + L\mu_D$$

$$SS_1 = Z_{1-\alpha}\sqrt{\sigma_D^2 L + \sigma_L^2\mu_D^2} + L\mu_D - (L+1)\mu_D$$

$$SS_1 = Z_{1-\alpha}\sqrt{\sigma_D^2 L + \sigma_L^2\mu_D^2} - \mu_D.$$

By using an order-up-to level of $\sum_{i=1}^{L+1} P_i + SS_1$, over $L+1$ weeks we will bring in enough material to cover the demand incurred in L weeks a percentage of the time equal to $(1 - \alpha) \times 100\%$.

Two-Week SRT

The probability that some demand is actually filled two weeks after its arrival is the probability that the order-up-to level over $L+1$ weeks covers demand over just $L - 1$ weeks.

Let X** denote the amount of demand in $L - 1$ weeks and let S_2 denote the order-up-to level appropriate for a two-week SRT. X** is normally distributed with mean $(L - 1)\mu_D$ and variance $\sigma_D^2(L - 1) + \sigma_L^2\mu_D^2$.

$$\text{Prob}\left(X^{**} \le S_2\right) = \text{Prob}\left(Z \le \frac{S_2 - (L - 1)\mu_D}{\sqrt{\sigma_D^2(L - 1) + \sigma_L^2\mu_D^2}}\right) = 1 - \alpha$$

This implies that

$$Z_{1-\alpha} = \frac{S_2 - (L - 1)\mu_D}{\sqrt{\sigma_D^2(L - 1) + \sigma_L^2\mu_D^2}}$$

$$S_2 = Z_{1-\alpha}\sqrt{\sigma_D^2(L - 1) + \sigma_L^2\mu_D^2} + (L - 1)\mu_D.$$

Since the POPLAN system will calculate order-up-to level as $S2 = \sum_{i=1}^{L+1} P_i + SS_2$. we have two expressions for the order-up-to level, S_2.

$$S_2 = (L + 1)\mu_D + SS_2 = Z_{1-\alpha}\sqrt{\sigma_D^2(L - 1) + \sigma_L^2\mu_D^2} + (L - 1)\mu_D$$

$$SS_2 = Z_{1-\alpha}\sqrt{\sigma_D^2(L - 1) + \sigma_L^2\mu_D^2} + (L - 1)\mu_D - (L + 1)\mu_D$$

$$SS_2 = Z_{1-\alpha}\sqrt{\sigma_D^2(L - 1) + \sigma_L^2\mu_D^2} - 2\mu_D.$$

General Case

In general, the safety stock required for a given SRT goal is given by:

$$SS = Z_{1-\alpha}\sqrt{\sigma_D^2(L + 1 - SRT) + \sigma_L^2\mu_D^2} - (SRT)\mu_D.$$

However, this equation only ensures arrival of material from the supplier no later than the SRT. It does not guarantee that the factory will actually have the final product built and ready for shipment to the customer no later than the SRT. Production cycle time must be incorporated into the equation to make the result useful in setting safety stocks to support product SRT objectives.

Let T_B denote the production cycle time required to build a week's worth of expected demand. Then

$$SS = Z_{1-\alpha}\sqrt{\sigma_D^2\left(L + 1 - SRT + T_B\right) + \sigma_L^2\mu_D^2} - \left(SRT - T_B\right)\mu_D.$$

Consider the three cases exhibited in Fig. 1. If we let build time be two weeks in all three cases and let SRT be 4, 2, and 0 weeks, respectively, then we have the following results.

Case A. $SS = Z_{1-\alpha}\sqrt{\sigma_D^2(L + 1 - 4 + 2) + \sigma_L^2\mu_D^2} - (4 - 2)\mu_D.$

Case B. $SS = Z_{1-\alpha}\sqrt{\sigma_D^2(L + 1 - 2 + 2) + \sigma_L^2\mu_D^2} - (2 - 2)\mu_D.$

Case C. $SS = Z_{1-\alpha}\sqrt{\sigma_D^2(L + 1 - 0 + 2) + \sigma_L^2\mu_D^2} - (0 - 2)\mu_D.$

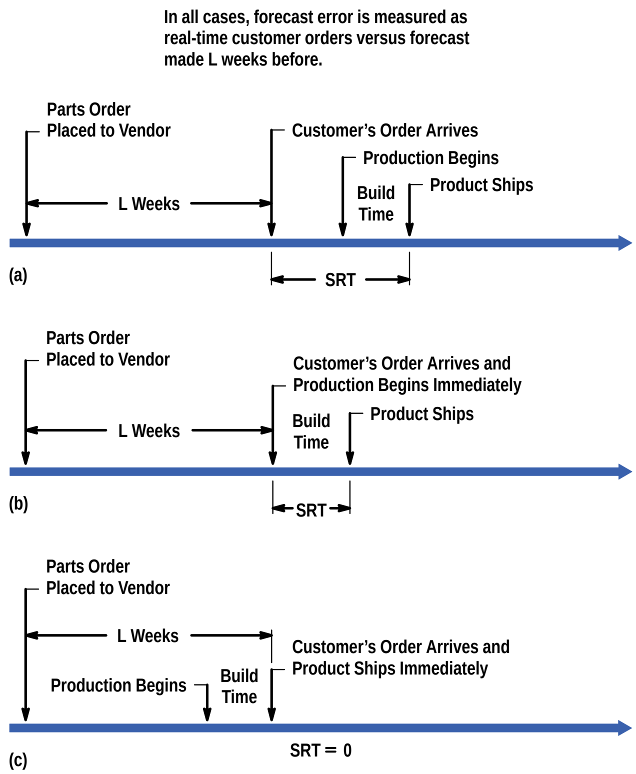

Fig. 1. *Production cycles for different SRT goals. (a) Case A: SRT = 4 weeks. (b) Case B: SRT = 2 weeks. (c) Case C: SRT = 0 weeks.*

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