Appendix III: The Expected Value and Variance of On-Hand Inventory when there Are Restrictions on Minimum Buy Quantities

Here we assume that restrictions on the size of orders placed to the supplier prevent procurement from ordering exactly the difference between the order-up-to level and the inventory position. The restriction in order size might be the result of a minimum buy size constraint placed by the supplier, a constraint that the order must be an integer multiple of a specified quantity, or the purchaser's desire that deliveries come at some delivery interval greater than weekly.

Let: Min = minimum order size constraint
Mult = multiple order size constraint
DI = desired delivery interval constraint.

Then the order size decision rule is given by:

$$\text{New order size} = M = k \times \text{Mult},$$

where k is the smallest integer such that:

1. $M \geq$ Order-up-to Level $-$ Inventory Position
2. $M \geq$ Min
3. $M \geq DI \times$ Average Weekly Demand.

Finally, we assume that the order is placed for the entire order quantity to be delivered L weeks later, that is, the order is not partitioned into pieces with separate delivery dates.

Let: I = On-hand physical inventory
S = Order-up-to level
Y = Amount of part consumed in first L weeks of the (L + R)-week cycle
C_S = Cycle stock = stock consumption to date during the R-week portion of the (L + R)-week cycle
SS = Safety stock
M = Order quantity
Δ = Increment above the order-up-to level S that the inventory position reaches as a result of having to order a quantity M.

$$I = (S + \Delta) - Y - C_S$$

$$E(I) = E(S) + E(\Delta) - E(Y) - E\left(C_S\right)$$

$$E(I) \cong \left(\sum_{i=1}^{L+R} P_i + SS\right) + E(\Delta) - \sum_{i=1}^{L} P_i - \frac{1}{2}\sum_{i=L+1}^{L+R} P_i$$

$$E(I) \cong SS + E(\Delta) + \frac{1}{2}\sum_{i=L+1}^{L+R} P_i.$$

To determine $E(\Delta)$ note that rather than buying strictly an amount equal to (S $-$ Inventory Position) we buy a quantity M. Therefore, the difference between what would be ordered without minimums and what is ordered with minimums varies between 0 and $M - 1$. We will assume that this difference is uniformly distributed within this range. Thus:

$$E(I) \cong SS + \frac{M-1}{2} + \frac{1}{2}\sum_{i=L+1}^{L+R} P_i.$$

The derivation of the variance of I is as follows.

$$V(I) = V(S) + V(\Delta) + V(Y) + V(C_S)$$

$$V(I) = 0 + V(\Delta) + V\left(\sum_{i=1}^{L} D_i\right) + V(C_S)$$

$$V(I) \cong V(\Delta) + \sigma_e^2 \mu_L + \sigma_L^2 \bar{P}_L^2 + \frac{1}{12}\left[R\sigma_e^2 + \left(\sum_{i=L+1}^{L+R} P_i\right)^2\right] + \frac{R\sigma_e^2}{4}$$

$$V(I) \cong \frac{(M-1)^2}{12} + \sigma_e^2 \mu_L + \sigma_L^2 \bar{P}_L^2 + \frac{1}{12}\left[R\sigma_e^2 + \left(\sum_{i=L+1}^{L+R} P_i\right)^2\right] + \frac{R\sigma_e^2}{4}$$

where $\bar{P}_L$ is the average of the plan over the L-week period immediately before the R-week period in question.

- Return to Article 6
- Go to Appendix IV
- Go to Next Article
- Go to Journal Home Page