Appendix II: The Expected Value and Variance of On-Hand Inventory when there Are no Restrictions on Minimum Buy Quantities

Let: I = On-hand physical inventory
S = Order-up-to level
Y = Amount of part consumed in first L weeks of the (L + R)-week cycle
C_S = Cycle stock = stock consumption to date during the R-week portion of the (L + R)-week cycle
SS = Safety stock

$$I = S - Y - C_S$$

$$I = \left(\sum_{i=1}^{L+R} P_i + SS\right) - \left(\sum_{i=1}^{L} D_i\right) - C_S$$

$$E(I) = E\left(\sum_{i=1}^{L+R} P_i + SS\right) - E\left(\sum_{i=1}^{L} D_i\right) - E(C_S)$$

$$E(I) \cong \sum_{i=1}^{E(L)+R} P_i + SS - \sum_{i=1}^{E(L)} P_i - E(C_S).$$

We will consider C_S to be uniformly distributed between 0 and $\sum_{i=L+1}^{L+R} D_i$. Thus,

$$E(I) \cong \sum_{i=1}^{E(L)+R} P_i + SS - \sum_{i=1}^{E(L)} P_i - \frac{1}{2}\sum_{i=E(L)+1}^{E(L)+R} P_i$$

$$E(I) \cong SS + \frac{1}{2}\sum_{i=E(L)+1}^{E(L)+R} P_i = SS + \frac{R\overline{P}_R}{2}.$$

The variance of I is derived as follows.

$$V(I) = V(S) + V(Y) + V(C_S)$$

Even though the P_i are not all fixed, and hence S changes every R weeks, S is still a constant with respect to the inventory result during the last R weeks of every (L + R)-week cycle. Hence, V(S) = 0.

$$V(I) = 0 + V\left(\sum_{i=1}^{L} D_i\right) + V(C_S)$$

$$V(I) \cong \left(\sigma_e^2 \mu_L + \sigma_L^2 \overline{P}_L^2\right) + V(C_S)$$

$$V(C_S) = E\left(V\left(C_S \middle| D_{L+1}, D_{L+2}, \ldots, D_{L+R}\right)\right) + V\left(E\left(C_S \middle| D_{L+1}, D_{L+2}, \ldots, D_{L+R}\right)\right)$$

$$E\left(C_S \middle| D_{L+1}, D_{L+2}, \ldots, D_{L+R}\right) = \frac{D_{L+1} + D_{L+2} + \ldots + D_{L+R}}{2}$$

$$V\left(E\left(C_S \middle| D_{L+1}, D_{L+2}, \ldots, D_{L+R}\right)\right) = V\left(\frac{D_{L+1} + D_{L+2} + \ldots + D_{L+R}}{2}\right)$$

$$= \frac{1}{4}V\left(\left(P_{L+1} + e_{L+1}\right) + \left(P_{L+2} + e_{L+2}\right) + \ldots + \left(P_{L+R} + e_{L+R}\right)\right)$$

$$= \frac{R\sigma_e^2}{4}$$

$$V\left(C_S \middle| D_{L+1}, D_{L+2}, \ldots, D_{L+R}\right) = \frac{\left(D_{L+1} + D_{L+2} + \ldots + D_{L+R}\right)^2}{12}$$

$$E\left(V\left(C_S \middle| D_{L+1}, D_{L+2}, \ldots, D_{L+R}\right)\right)$$

$$= E\left[\frac{\left(D_{L+1} + D_{L+2} + \ldots + D_{L+R}\right)^2}{12}\right] = \frac{1}{12}E\left(G^2\right),$$

where $G = D_{L+1} + D_{L+2} + \ldots + D_{L+R}$.

$$E\left(G^2\right) = \left(\sigma_G^2 + \mu_G^2\right) = \left[R\sigma_e^2 + \left(\sum_{i=L+1}^{L+R} P_i\right)^2\right]$$

$$E\left(V\left(C_S \middle| D_{L+1}, D_{L+2}, \ldots, D_{L+R}\right)\right) = \frac{1}{12}\left[R\sigma_e^2 + \left(\sum_{i=L+1}^{L+R} P_i\right)^2\right]$$

$$V\left(C_S\right) = \frac{1}{12}\left[R\sigma_e^2 + \left(\sum_{i=L+1}^{L+R} P_i\right)^2\right] + \frac{R\sigma_e^2}{4}$$

Hence,

$$V(I) \cong \sigma_e^2\mu_L + \sigma_L^2\bar{P}_L^2 + \frac{1}{12}\left[R\sigma_e^2 + \left(\sum_{i=L+1}^{L+R} P_i\right)^2\right] + \frac{R\sigma_e^2}{4},$$

where $\bar{P}_L$ is the average of the plan over the L-week period immediately before the R-week period in question.

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