Appendix I: Mathematics of Production and Material Planning for the Simple Model

I-1 The Planning Function

The planning function is actually an analytic model embedded within a discrete event simulation model. The fundamental principle on which the production and material planning algorithms are based is the conservation of mass, that is, consumption cannot be higher than the total supply available. The order in which the build plan computation is done is the reverse of the order in which subassemblies are built and products are shipped (i.e., from shipment to product build to part order). For ease of explanation, the current week is considered to be week 0. This derivation emphasizes clarity of explanation rather than rigorous detail.

There are three sets of decision variables to be determined for each week: *s(t)*, the shipment plan, *b(t)*, the build plan, and $m_j(t)$, the material ordering plan. These are shown in italics.

Before we get into the mathematical formulation, let us first look at the process of computation. Fig. 1 illustrates how the production and material planning algorithms work in this model. The computational process is described in the following order:

- I-2 describes the notation shown in Fig. 1.
- I-3 describes the safety stock computation.
- I-4 describes the initial conditions for computation.
- I-5 describes the computation of the shipment plan.
- I-6 describes the computation of the build plan.
- I-7 describes computation of the number of units started this week.
- I-8 describes the computation of the material consumption and material ordering plans.
- I-9 describes the actual material ordered this week.
- I-10 describes the computation of the number of weeks for each of the plans.

I-2 Notation

- n, s, t = indexes for week number (current week = 0)
- f(t) = Current forecast of product orders for week t, t = 0, 1, …, N_f
- F(t) = FGI at end of week t
- W(t) = WIP at end of week t
- B(t) = Backlog units at end of week t
- B(t,s) = Backlog units at end of week t having shipment dates in week s
- *s(t)* = Planned shipments during week t
- *b(t)* = Units planned to be started during week t
- B = Build time in number of weeks
- Y = Quoted availability in number of weeks
- S = Shipment or transit time
- j = Index relating to part
- Q_j = Quantity of part j per unit of product
- $q_j(t)$ = Planned consumption of part j during week t

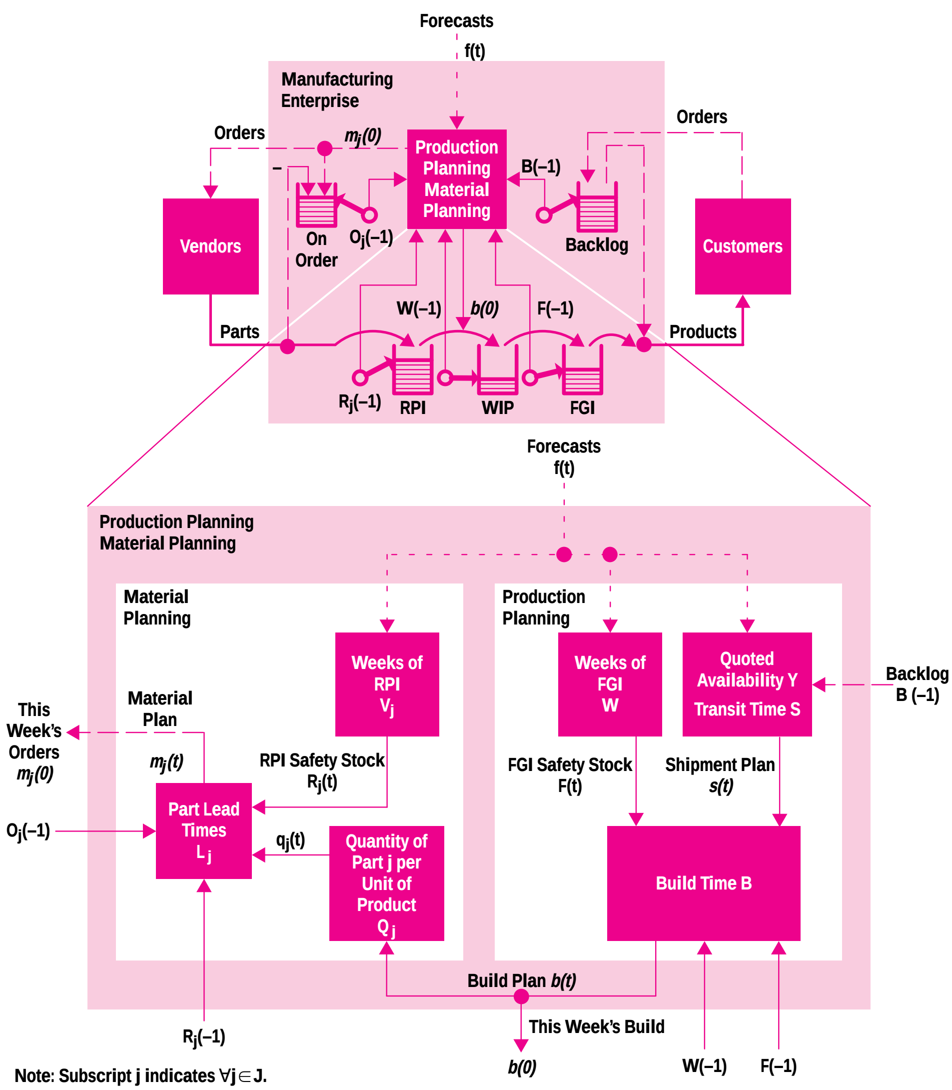

Fig. 1. Notation and production/material planning. The shipment plan is computed from the backlog, forecasts, quoted availability, and transit time. The build plan is computed from the shipment plan, the build time, WIP, FGI, and FGI safety stock. The actual build is computed from the build plan and the material availability. The material consumption plan is computed from the build plan and the bill of materials. The material ordering plan is computed from RPI, RPI safety stock, the material consumption plan, on-order material, and lead time.

- $m_j(t)$ = Planned quantity of material j to be ordered during week t, t = 0, 1, …, N_j, $j \in J$
- $R_j(t)$ = RPI of part j at the end of week t
- $r_j(t)$ = Units of part j received during week t
- $O_j(t)$ = Units of part j on order at the end of week t
- L_j = Vendor lead time for part j
- J = Set of parts that go into the product
- w = FGI safety stock in weeks of demand
- V_j = RPI safety stock of part j in weeks of demand
- N_s = Last week for computing shipment plan
- N_b = Last week for computing build plan
- N_f = Last week used for forecasts
- N_j = Last week for computing material order for part j.

Since the current week is 0, the values of these variables represent actual values for weeks before 0, and the values are computed, set, or derived for weeks 0 and later. In particular, the values of variables at the end of week –1 represent the current values of those variables, as described in I-4. All numerical quantities except time indexes are zero or positive.

I-3 Safety Stock Computation

Safety stock is expressed in number of weeks of 13-week leading average forecast. The 13-week leading average forecast at the end of week t is defined as:

$$\overline{f(t)} = \frac{1}{13}\sum_{i=1}^{13} f(t+i) \qquad (1)$$

The target FGI safety stock at the end of week t is w weeks and the target RPI safety stock at the end of week t for part j is V_j weeks. The expressions for these quantities are:

$$F(t) = w\overline{f(t)} \qquad (2)$$

$$R_j(t) = V_j Q_j \overline{f(t)} \qquad (3)$$

I-4 Initial Conditions

- F(–1) = Actual FGI at the end of the previous week, that is, current FGI
- W(–1) = Actual WIP at the end of the previous week, that is, current WIP
- $O_j(-1)$ = Actual part j on order at the end of the previous week, that is, current on-order material
- $R_j(-1)$ = Actual RPI for part j at the end of the previous week, that is, current RPI for part j.
- B(–1) = Order backlog in units at the end of the previous week, that is, current backlog:

$$B(-1) = \sum_{s \in \langle \text{all shipment dates in current backlog} \rangle} B(-1, s) \qquad (4)$$

- B(–1,s) = Component of current backlog with shipment date in week s.

I-5 Shipment Plan

The shipment plan indicates prospective shipments during the current and future weeks. It is computed on the assumption that customer orders are not shipped before they are due, but are shipped in time to satisfy the quoted availability requirements. This implies that for any week, the orders planned to be shipped are those that are already late (i.e., should have been shipped in an earlier week) and those that must be shipped to be delivered on time. Notice that in computing the shipping plan, we do not take into account the amount of inventory on hand or in process. This is representative of the way shipment plans are computed and then subsequently checked against reality.

Put another way, this can be expressed as planning to ship the minimum quantity in each week that will satisfy the quoted availability criteria. The problem can be formulated as shown in the set of equations below, which indicate that we are attempting to minimize shipments in the current week, current plus next week, current plus next 2 weeks, and so on such that the total shipments in those weeks is greater than the current existing backlog whose shipment date is already past or in those weeks, plus the forecasted orders whose desired shipment dates lie in those weeks.

Minimize *s(n)*, n = 0,1,…,N_s

$$\text{such that } \sum_{t=0}^{n} s(t) \ge \sum_{t \in \langle i \mid i \le n \rangle} B(-1, t) + \sum_{t=0}^{n-(Y-S)} f(t)$$

and $s(n) \ge 0$.

These equations define a series of (N_s + 1) linear programming problems. However, this formulation will always return a set of feasible solutions, and the optimal feasible solutions can be expressed in closed form as follows:

$$s(n) = \begin{cases} \sum_{s \in \langle i \mid i \le 0 \rangle} B(-1, s) & \text{for } n = 0 \\ B(-1, n) & \text{for } 0 < n < Y - S \\ f(n - (Y - S)) & \text{for } n \ge Y - S. \end{cases} \qquad (5)$$

The term (Y – S) is the difference between the quoted availability and the transit time (i.e., the order-to-ship time to achieve on-time delivery), and indicates the time in the future after which shipments depend solely on forecasts.

I-6 Build Plan

The build plan, which indicates how many units are to be started in the current week 0 and succeeding weeks, is based on the assumption that the FGI levels at the end of weeks 0,1,…,B–1 have already been determined by the current FGI, WIP, and shipments preceding week 0. It further assumes that we might be able to control FGI at the end of week B or later by deciding how many units we start this week and future weeks, that is, by controlling *b(0),b(1),…,b(n).* We want to keep the *b(n)* as low as possible but greater than or equal to 0, such that the total planned build during weeks 0 through n must be greater than or equal to shipments during weeks 0 through B+n plus FGI at the end of week B+n minus current FGI and WIP. The complete formulation is as follows:

Minimize *b(n)*, n = 0,1,…,N_b

$$\text{such that } \sum_{t=0}^{n} b(t) \ge \sum_{t=0}^{B+n} s(t) + F(B+n) - F(-1) - W(-1)$$

and $b(n) \ge 0$.

Again, the above is a series of (N_b+1) linear programming problems, with optimal feasible solutions that are expressed in closed form as follows:

$$b(n) = \max\left\{0,\ F(B+n) + \sum_{t=0}^{B+n} s(t) - F(-1) - W(-1) - \sum_{t=0}^{n-1} b(t)\right\}, \qquad (6)$$

for n = 0, 1, …, N_b.

To summarize the above, the current build plan should look as follows:

Week:	0	1	2	…	n
Planned Build:	*b(0)*	*b(1)*	*b(2)*	…	*b(n).*

I-7 Actual Units Started

The actual units started this week, b_0, will be *b(0)* if there is sufficient material. If there is insufficient material the actual units started is the maximum possible with the available material, or:

Maximize b_0

such that $Q_j b_0 \le R_j(-1) + r_j(0)$, $\forall j \in J$

and $0 \le b_0 \le b(0)$,

for which the closed form solution is:

$$b_0 = \min\left\{ b(0), \min_{j \in J}\left(\frac{R_j(-1) + r_j(0)}{Q_j}\right)\right\}. \qquad (7)$$

I-8 Material Requirement Analysis

If the lead time for a part j is L_j weeks, the RPI level for part j at the end of weeks 0,1, … L_j–1 has been determined by material on hand, material on order, and projected use. We could control RPI for part j at the end of week L_j or later by deciding how much of part j we order in this week and subsequent weeks. The estimated material consumption during a week is the quantity of the material for the build for that week, that is:

$$q_j(t) = Q_j b(t). \tag{8}$$

The material ordered during weeks 0 through n must be greater than or equal to the material consumed during weeks 0 through L_j+n plus the desired safety stock at the end of week L_j+n minus the current on-hand material and the current on-order material. This can be expressed mathematically as follows:

Minimize $m_j(n)$, n = 0,1,…,N_j, j∈J

such that $\sum_{t=0}^{n} m_j(t) \geq \sum_{t=0}^{L_j+n} q_j(t) + R_j(L_j + n) - R_j(-1) - O_j(-1)$

and $m_j(n) \geq 0$.

After substituting equation 8, this becomes a series of linear programming formulations for which the closed form solution is:

$$m_j(n) = \max \begin{cases} 0 \\ \left(Q_j \sum_{t=0}^{L_j+n} b(t) + R_j(L_j + n) - R_j(-1) - O_j(-1) - \sum_{t=0}^{n-1} m_j(t) \right) \end{cases} \tag{9}$$

for n = 0, 1, …, N_j, j∈J.

The current material ordering plan is shown by the following table.

	Week				
	0	**1**	**2**	…	**n**
Material 1	$m_1(0)$	$m_1(1)$	$m_1(2)$	…	$m_1(n)$
Material 2	$m_2(0)$	$m_2(1)$	$m_2(2)$	…	$m_2(n)$
…	…	…	…	…	…
Material j	$m_j(0)$	$m_j(1)$	$m_j(2)$	…	$m_j(n)$
…	…	…	…	…	…

I-9 Actual Material Ordered

Given the table above, the actual material ordered in this week must be $m_j(0)$, ∀j∈J.

I-10 Determination of the Required Number of Weeks

Since we want to compute the material procurement plan for material j for periods 0 through N_j, we need to make sure we have values of the forecasts, shipment plan, and build plan far enough in the future to allow us to do so. This section shows how many periods of those plans we need to compute.

In 10 through 16 below, "$m_j(n)$ requires x(n)" should be read as, "Computing $m_j(n)$ requires values of x(0), x(1), …, x(n)." Thus 10 should be read as, "Computing $m_j(N_j)$ requires the values of $R_j(0)$, $R_j(1)$, …, $R_j(L_j+N_j)$."

From 9,

$$m_j(N_j) \text{ requires } R_j(L_j + N_j) \tag{10}$$

$$\text{and } m_j(N_j) \text{ requires } b(L_j + N_j). \tag{11}$$

From 10, 3, and 1,

$$m_j(N_j) \text{ requires } f(L_j + N_j + 13). \tag{12}$$

From 11 and 6,

$$m_j(N_j) \text{ requires } F(B + L_j + N_j) \tag{13}$$

$$\text{and } m_j(N_j) \text{ requires } s(B + L_j + N_j). \tag{14}$$

From13, 2, and 1,

$$m_j(N_j) \text{ requires } f(B + L_j + N_j + 13). \tag{15}$$

From 14, 5, and 1,

$$m_j(N_j) \text{ requires } f(B + L_j + N_j - (Y - S)). \tag{16}$$

Computation of N_b. From 11,

$$N_b = \max_{j \in J} \{L_j + N_j\}. \tag{17}$$

Computation of N_s. From 14,

$$N_s = \max_{j \in J} \{B + L_j + N_j\}. \tag{18}$$

Computation of N_f. From 12, 15, and 16,

$$N_f = \max_{j \in J} \begin{cases} L_j + N_j + 13 \\ B + L_j + N_j + 13 \\ B + L_j + N_j - (Y - S) \end{cases} \tag{19}$$

Since B ≥ 0, (Y – S) ≥ 0, the middle expression dominates, and 19 reduces to:

$$N_f = \max_{j \in J} \{B + L_j + N_j + 13\}. \tag{20}$$

Appendix II: Weekly Event Sequence

In the following table, periodically scheduled events are shown in sequence.

Event Time	Event Frequency	Initiators	Event Description
Monday 1:00	Weekly	Customers	Generate and send orders; these orders are received by the Adder factory at 9:30:00 the same day.
Monday 8:00	Weekly	Factory	Completes computing FGI safety stock for future weeks. Completes computing shipment plan and build plans.
Monday 9:00	Weekly	Factory	Completes computing material requirements plan. Completes computing material procurements plan.
Monday 10:00	Weekly	Factory	Generates current week's material orders. Material orders arrive at the vendors instantaneously.
Monday 10:00:01	Weekly	Vendors	Finish filling and shipping orders due this week. Shipments arrive at the factory instantaneously.
Monday 10:30	Weekly	Factory	Begins current week's production. Completes production started two weeks ago.
Friday 16:30	Weekly	Factory	Completes filling and shipping orders for the week.
Friday 23:58	Weekly	Simulation Executive	Records values of all the state variables.

Appendix III: Details of Part Commonality Experiments

The following table shows the definitions used to describe part commonality. MC stands for material cost, with uppercase denoting dollar values and lowercase denoting percentage values. m represents the set of material.

	Set of Material	Value of Material	Percentage Value
Common to products i and i–1	$m_{i,i-1}$	$MC_{i,i-1}$	$mc_{i,i-1} = \frac{MC_{i,i-1}}{MC_i} \times 100$
Unique to product i	$m_{i,i}$	$MC_{i,i}$	$mc_{i,i} = \frac{MC_{i,i}}{MC_i} \times 100$
Common to products i and i+1	$m_{i,i+1}$	$MC_{i,i+1}$	$mc_{i,i+1} = \frac{MC_{i,i+1}}{MC_i} \times 100$

Commonality occurs only between adjacent products. This implies that a part can be used in at most two products.

Each of the $MC_{i,j}$ is further broken up into class A, B, and C parts with relative values 50, 30, and 20 percent. Each of these classes is made up of 6, 10, and 14 week lead times with relative values 25, 40, and 35 percent. (See Table I on page NO TAG.)

At the end of the product i life cycle, obsolete inventory (if any) should come only from parts in sets $m_{i,i}$ and $m_{i,i-1}$. Any leftover parts from $m_{i,i+1}$ can be used in product i+1. This implies that $mc_{i,i-1}$ and $mc_{i,i}$ impact the obsolete inventory at the end of the product life cycle for product i.

The values shown in the following table should be derived from the real bill of materials. For our experiments, we reverse the process, that is, we generate a bill of materials from the table, which was generated heuristically from the experimental scenarios, with the following constraints on the values of mc:

- For each i and j, $mc_{i,j}$ must be greater than or equal to 0 and less than or equal to 100.
- For each i, the sum of $mc_{i,j}$ over all j must be 100.
- In each experiment, if any $mc_{i,i+1}$ is zero, then $mc_{i+1,i}$ must also be zero.

Description of Experimental Scenarios

Run M-0: no part commonality at all between adjacent products.

Run M-1: 20% part commonality between adjacent products. The parts common to products i and i+1 make up 20% of the part values of both products. This may happen by a reduction in either part quantity or part cost, but the reason is not reflected in the dollar value of leftover inventory or material.

Run M-2: 20% part commonality when moving to a new product. The parts common to products i–1 and i make up 20% of the part value of product i; the rest of the value of product i is split equally between the parts unique to product i and those common to products i and i+1. Since product Adder-1 has no prior product, the value is split equally between unique parts and parts common to Adder-1 and Adder-2. 20% of the value of Adder-2 is made up of parts common to Adder-1 and Adder-2; the remaining 80% is split equally between unique parts and parts common to Adder-2 and Adder-3. 20% of the value of product Adder-4 is made up of parts common to Adder-3 and Adder-4; the balance of the value is unique parts since there are no succeeding products.

Run M-3: 50% and 25% part commonality between alternate products. There is 50% part commonality between products Adder-1 and Adder-2 and between Adder-3 and Adder-4; there is 25% part commonality between Adder-2 and Adder-3.

Run M-4: 50% part commonality between adjacent products; no unique parts in Adder-2 and Adder-3; 50% unique parts in Adder-1 and Adder-4.

Run M-5: 80% part commonality between succeeding products.

Part Commonality Data (%) for Multiple Product Crossover

i	Product	Demand (units)	Product Cost ($)	Common Parts (%)	Experiment Run					
					M-0	M-1	M-2	M-3	M-4	M-5
1	Adder-1	V	10,000	$mc_{1,1}$	100	80	50	50	50	20
				$mc_{1,2}$	0	20	50	50	50	80
2	Adder-2	1.3V	$0.85 \times 10{,}000$	$mc_{2,1}$	0	20	20	50	50	80
				$mc_{2,2}$	100	60	40	25	0	10
				$mc_{2,3}$	0	20	20	25	50	10
3	Adder-3	$1.3 \times 1.3V$	$0.85 \times 0.85 \times 10{,}000$	$mc_{3,2}$	0	20	20	25	50	80
				$mc_{3,3}$	100	60	40	25	0	10
				$mc_{3,4}$	0	20	40	50	50	10
4	Adder-4	$1.3 \times 1.3 \times 1.3V$	$0.85 \times 0.85 \times 0.85 \times 10{,}000$	$mc_{4,3}$	0	20	20	50	50	80
				$mc_{4,4}$	100	80	80	50	50	20

Appendix IV: Details of Explanations for Experiments 0 and 1a

IV-1 Estimated Financial Impact Based on Theoretical Considerations for Experiment 0

The impact of product Adder on the financial situation of the enterprise, as explained on page NO TAG, is:

- Total PCFT = $7,800,000
- Mature volume = MV = mature PCFT = $800,000/month or $200,000/week
- Consignment inventory = $300,000.

IV-2 Mature Demand Week Considerations for Experiment 0

RPI Material to Support Mature Demand

	Class A	Class B	Class C	All Classes
① Percentage of Part Value in Product	50%	30%	20%	100%
② Weekly Use during Mature Demand ① × MV	$100k	$60k	$40k	$200k
③ RPI Safety Stock in Weeks	4	8	16	N/A
④ RPI in $: ③ × MV	$400k	$480k	$640k	$1520k
⑤ RPI in Weeks of MV ④ ÷ MV	2	2.4	3.2	7.6

On-Order Material to Support Mature Demand

① Lead Time	6 weeks	10 weeks	14 weeks	All Parts
② Percentage of Part Value in Product	25%	40%	35%	100%
③ Weekly Order during Mature Demand ② × MV	$50k	$80k	$70k	$200k
④ Amount on Order = Weekly Order × Lead Time: ③ × ①	$300k	$800k	$980k	$2080k
⑤ Percent Value of Part on Order: ④ ÷ $2080k	14.4%	38.5%	47.1%	100%
⑥ On-order Material in Weeks of MV ④ ÷ MV	1.5	4.0	4.9	10.4

Total Inventory Metrics during Mature Demand

	Weeks of Mature Demand	Dollars
① RPI	7.6	$1520k
② WIP	2.0	$400k
③ FGI	2.0	$400k
④ On-Hand Inventory: ① + ② + ③	11.6	$2320k
⑤ On-Order Material	10.4	$2080k
⑥ Committed Inventory: ④ + ⑤	22.0	$4400k
⑦ Consignment Inventory	1.5	$300k
⑧ Total Committed Inventory: ⑥ + ⑦	23.5	$4700k

IV-3 End-of-Life Considerations for Experiment 0

Total PCFT = $7,800,000. Net profit = $78,000(i/100), where i is the profit as a percent of PCFT.

The following table summarizes the impact on the profitability of various margins i.

Write-Off as a Function of Profit on Shipped Units

① Profit Margin i	5%	10%	20%	30%
② Profit from Trade Units $7.8M × ①	$390k	$780k	$1560k	$2340k
③ Leftover Material		$64,615		
④ Leftover Material as % of Net Profit: ③ ÷ ②	16.57%	8.28%	4.14%	2.76%
⑤ Consignment		$300,000		
⑥ Consignment as % of Net Profit ⑤ ÷ ②	76.92%	38.46%	19.23%	12.82%
⑦ Total EOL Material as % of Net Profit: (③ + ⑤) ÷ ②	93.49%	46.75%	23.37%	15.58%

The following table shows the impact on Class C EOL material of reducing safety stock levels. These results were computed using means other than simulation.

Weeks of Class C Safety Stock	Class C EOL Material
16 weeks	$64,615
15 weeks	$35,385
14 weeks	$13,846
13 weeks	$0

IV-4 Why There Is Class C material Left Over for Experiment 0

The last period in which we expect to receive orders is week 68. The end of week 55 is 13 weeks before the end of the product life cycle. From the Adder order forecast in Fig. 2 on page NO TAG and the target RPI safety stock for class C material being 16 weeks of the 13-week leading average forecast (Table Ib on page NO TAG), at the end of week 55 the amount of class C material in RPI should theoretically be 16/13 of the total demand to the end of life, or (16/13) × (1¾ × V) = (28/13) × V units, where V = 80.

In week 56, we need to start building the units for orders received in week 55. Ignoring the current FGI, the maximum new build from week 56 to the the end of life is equal to the demand from week 55 through the end of life, that is, 2V. Thus, at the end of week 55, there is more class C material on hand—enough to build (28/13) × V units—than needed for the demand to the the end of the product life cycle.

Remember that we did not consider units in FGI. If we want to reduce FGI units down to 0 by the end of the product life cycle, the total new build must be less than that computed above, and hence there will be even more class C material left over.

In summary, one reason for the leftover class C material is that the safety stock computation requires holding more class C raw material in RPI 13 weeks before the end of life than can be consumed by orders received in the last 14 weeks of the product life cycle.

IV-5 Why Orders Cannot Be More than 14 Weeks Late for Experiment 1a

Assume that an order comes in during week x. In the worst case we have not yet ordered any material for the unit that goes with this order. The earliest the material can be ordered is week x+1, and the longest lead time part will be delivered during week (x+1)+14, which is week x+15. Since build time is 2 weeks, the unit is ready in week x+17. Since transit time is 1 week, the unit is delivered to the customer in week x+18. Since the quoted availability is 4 weeks, on-time delivery means the customer should receive it in week x+4. This means that the lateness is 14 weeks.