New Capabilities in Digital Low-Frequency Spectrum Analysis

A new Fourier analyzer and two fast-transform peripherals adapt to a wide range of applications.

By Stephan G. Cline and Norman D. Marschke

SPECTRUM ANALYSIS, a powerful technique of signal and system analysis, is the process of resolving a physical waveform into sinusoidal components having appropriate phase relationships. That this can be done was shown by J.B.J. Fourier in the 18th century. The reason for its great utility is that a simple statement of spectrum content or bandwidth often makes possible engineering judgments about a problem that couldn't be made on the basis of observations of the waveform itself.

In spite of its usefulness, applications of spectrum analysis were concentrated for many years in the radio and electronic sciences, where the frequencies encountered are in the megahertz range and higher. Now, however, the use of spectrum analysis at low and moderate frequencies is on the increase, partly because many more people realize its power as an engineering tool, but mainly because accurate and stable instrumentation has recently become available for this frequency range. Problems in mechanical vibrations, underwater sound, biomedicine, communications, machine maintenance, feedback control systems, speech, and acoustics are now yielding to spectrum analysis. The instruments being used are mostly digital analyzers which compute spectra by means of one form or another of the fast-Fourier-transform (FFT) algorithm.[1]

Within the last year, faster computers and hardware FFT processors have significantly increased the speed of Fourier analysis, to the point where real-time spectrum analysis to beyond 10 kHz can be done using entirely digital instruments. Among Hewlett-Packard's second-generation Fourier-analysis instruments are a new Fourier Analyzer, a new plug-in FFT Arithmetic Unit for HP computers, and a new peripheral Fast Fourier Processor.

Why Digital?

Why did the use of spectrum analysis at low frequencies have to wait for the development of digital instruments before becoming widespread? The answer can be seen by comparing digital analyzers with previously available instruments.

The simplest way to resolve a waveform into its spectral components is to use a narrow bandpass filter and a voltmeter. Measuring the portion of the signal passed by the filter gives an approximation to the size of the sinusoidal component at the filter center frequency. If the filter is tunable, spectrum analysis can be carried out over a range of frequencies. This is the principle of the wave analyzer, the most elementary spectrum analyzer.

The wave analyzer has the basic limitation of all spectrum analyzers. The bandwidth of the filter, and therefore its resolution, determines its response time. A period of time approximately equal to the reciprocal of the analyzer passband must be spent at each frequency to get an accurate estimate of the spectral content at that frequency. This isn't a serious problem at frequencies in the megahertz range and higher, but at low frequencies, where the resolution needed may be on the order of one hertz, a single-frequency measurement may take as much as one second.

If the signal being analyzed is random, each reading must be averaged over an even longer period of time to get a statistically stable or smooth result. A measurement made in this way may correctly measure the spectrum at 1.0 kHz while missing a short term event at 1.5 kHz.

For these reasons, wave analysis is suitable only when the signal being analyzed is stationary and when comparatively long periods of time are available to sweep over the band of interest. Heterodyne

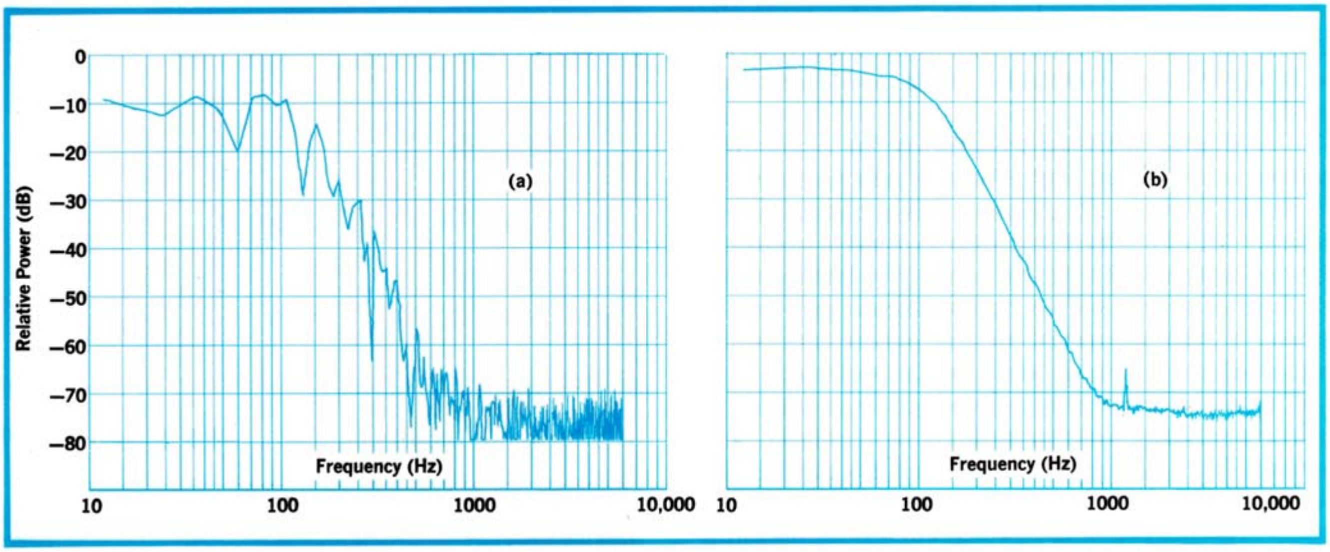

Fig. 1. *Power spectra of random noise illustrate resolution, dynamic range, and flexibility of digital analyzers. (a) Spectrum computed from one 102.4 ms record. (b) Average of 200 records has same 512-channel resolution as single record. Dynamic range is 80 dB. Note 1.2-kHz component at about −70 dB which is not visible in unaveraged spectrum.*

techniques which translate the measurement to higher frequencies make for better accuracy, better display, and less cost, but they don't solve the time-bandwidth limitation at low frequencies. Spectrum analyzers of this type are still basically single-filter wave analyzers.

The measurement time-bandwidth problem can only be alleviated if some form of parallel filtering is employed. The most direct way to build a spectrum analyzer that is real-time at all frequencies is to use a set of parallel filters. Such a method is most effective when filters of constant percentage bandwidth, such as ⅓ octave or full octave, are employed. Octave-band analyzers of this type are very useful and effective instruments.

Speed-Up Analyzer

However, when higher resolution or constant-bandwidth filters are needed, this approach has proved impractical. The nearest solution to parallel filtering using constant-bandwidth, narrow-band filters is a digital-analog hybrid approach called the speed-up analyzer. In this analyzer an analog-to-digital converter samples the waveform to be analyzed at a rate F_s and loads a memory with a record of N samples over a period of time $T = N/F_s$. The data gathered in this way is analyzed by recirculating it in the memory at a much higher rate. The data time scale is thus reduced and the speeded up digital data is applied to a digital-to-analog converter, resulting in a new signal with scaled-up frequency. This new signal is then analyzed by the equivalent of a high-frequency wave analyzer. At the higher frequency the analyzer sweep time is greatly reduced and near-real-time analysis can be carried out*. It isn't quite real-time, though, because there's a 20-to-100-millisecond gap between the processing of one T-second record and the processing of the next, so some of the signal information is lost.

All-Digital Analyzer

An important limitation on the performance of speed-up analyzers is the loss of dynamic range and resolution introduced by the analog circuitry of the final wave analyzer. Better dynamic range and resolution and greater flexibility in data processing can be obtained with a purely digital analyzer.

Like the speed-up analyzer, the all-digital analyzer samples the input signal at a rate F_s for T seconds and loads $N = TF_s$ sample values into its memory. Certain measurement limits imposed by the sampling process are shared by the digital analyzer and the speedup analyzer. First, the highest frequency that can be analyzed without ambiguity is $1/2\ F_s$. This maximum frequency is the Nyquist folding frequency; above it, aliasing error occurs[2]. Second, the resolution is determined by the record length. A record length of T seconds implies that the frequency resolution is $\Delta f = 1/T$.

This smallest frequency element is a result of the fact that for a record of length T seconds no discrete

*Real-time analysis is defined here as the analysis of contiguous records of data with no gaps.

Fourier transform can be defined with a resolution narrower than Δf Hz. It has nothing to do with any other hardware constraint such as filter bandwidths.

The digital analyzer transforms the N digitized values stored in its input memory to a set of N/2 Fourier coefficients spaced 1/T Hz apart. The result of this Fourier transform is a set of real and imaginary coefficients, which may be used directly or further processed to obtain the magnitude and phase of the spectrum, or the power (magnitude squared) spectrum, or many other functions.

The measurement time-bandwidth limitation of wave analyzers isn't a problem with digital analyzers. From a single T-second sample record of N samples, the digital analyzer computes a complete spectrum equivalent to N/2 wave-analyzer measurements each 1/T Hz apart. The same spectrum would take at least N/2 times as long to measure with a wave analyzer.

If the data being analyzed is random or has a random component, so that several separate sample records must be averaged to smooth the result, this is easily done in digital spectrum analysis by summing the results of one sample record into the accumulated result from past samples. One of the advantages of digital computation of the spectrum is that averaging may be done digitally without the use of analog integrators, which have limited dynamic range and linearity.

An Example

Fig. 1 illustrates some of the capabilities of the latest digital analyzers. It shows the power spectrum of a band-limited random noise signal computed in real time. Each record used to compute the spectrum is 102.4 milliseconds long and consists of 1024 samples of the noise signal.

Fig. 1(a) is the spectrum computed from one record, and the variance caused by the randomness of the signal is evident. Fig. 1(b) is the average of the spectra computed from 200 sample records. Besides being much smoother, this spectrum reveals the presence of a periodic component at 1.2 kHz. These results, computed from only 20.48 seconds of data, are good examples of the resolution, dynamic range, and processing flexibility of a digital analyzer.

New Fourier Analyzer

Fig. 1 was produced by the new Model 5451A Fourier Analyzer, Fig. 2. Like earlier HP Fourier analyzers[3], the new analyzer is a keyboard-controlled computer-based system capable of sampling one or more input signals and computing power spectra, cross power spectra, transfer functions, coherence functions, correlation functions, and many other functions. These computations and other complex data manipulations are performed in the computer by software routines which are initiated by pushing buttons on the keyboard. The user can choose to add his own routines to the software supplied. He can also use the integral minicomputer separately as a general purpose computer. Fig. 3 is a block diagram of the Fourier analyzer system.

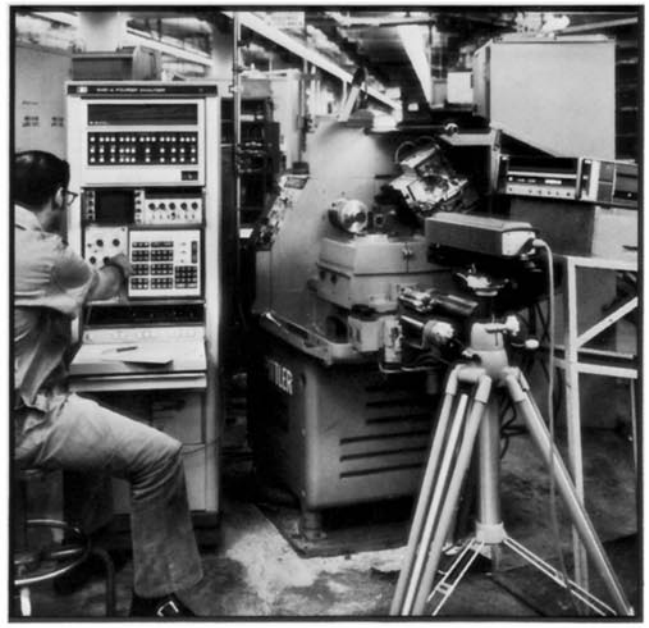

Fig. 2. *New Model 5451A Fourier Analyzer, a minicomputer-based, keyboard controlled, dual-channel analyzer, can do spectrum analysis, correlation, coherence analysis and many other complex data manipulations. New capabilities are 80-dB dynamic range, relocatable software, buffered input, and a faster computer.*

Differences between the new 5451A and earlier models are a new faster computer, Model 2100A[4], greater dynamic range (80 dB for both amplitude and power spectrum), relocatable software which simplifies adding and modifying user-written programs, and a buffered input mode which improves processing efficiency.

Fig. 4 shows the difference between the new buffered input mode and the ordinary input mode. Ordinarily, an input record is read into the computer memory and then processed, and while the power spectrum is being computed no new input data is collected. Without its optional hardware processors, the 5451A Fourier Analyzer may take as long as 1.5 seconds to complete these processing steps for a 1024-point record. Considerable improvement is realized when the buffered input mode of operation is used. In this mode data is read into a buffer

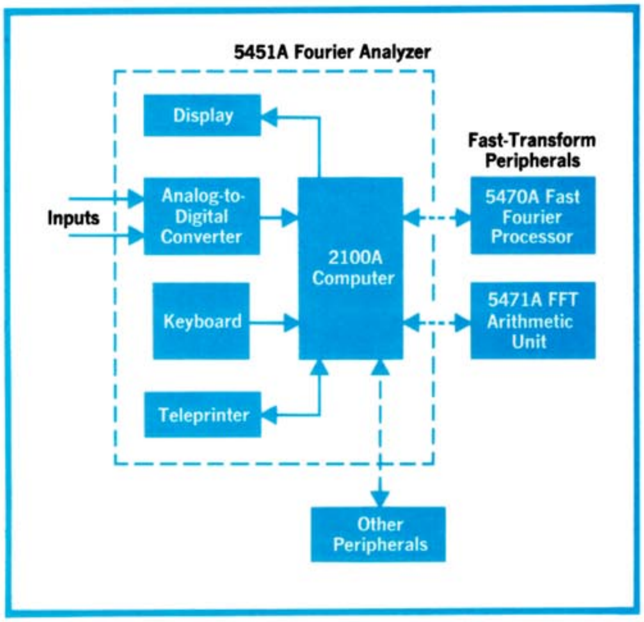

Fig. 3. *Analog-to-digital converter samples input signals at rates to 50 kHz. Keyboard controls data manipulations done by software in basic 5451A Fourier Analyzer. Fast-processor options add hardware transform capability.*

block while the previous record is being processed. With this parallel input-while-processing scheme it is possible to achieve real-time spectrum analysis to 300 Hz using a 5451A Fourier Analyzer with only a software Fourier transform.

Faster Fourier Analysis

Further improvements in processing speed are realized with special-purpose hardware which speeds up the required computations. Either the 5470A Fast Fourier Processor or the 5471A Fast Fourier Transform Arithmetic Unit can be added as options to the 5451A Fourier Analyzer for this purpose.

Increases in processing speed increase the analyzer's upper frequency limit for real-time analysis. Increased speed also saves machine time, which is important in test situations of limited duration, such as vibration testing of a spacecraft which might be fatigued in a long test.

The 5470A Fast Fourier Processor (Fig. 5) is an external peripheral which has its own data memory and function control memory. It takes only 15 milliseconds to compute a Fourier transform for a 1024-point block size, compared to one second for the standard 5451A Fourier Analyzer.

The four-card 5471A FFT Arithmetic Unit (Fig. 6) fits into four contiguous I/O slots in the computer and speeds up computations while continuing to use the computer for data storage and function control. With the 5471A a 1024-point transform can be computed in 160 ms.

The software drivers for the 5470A and 5471A are written so analyzer operation is exactly the same as the standard 5451A Fourier Analyzer, except that the execution times are much faster.

The following array operations are performed by the 5470A or the 5471A options in the 5451A.

- Forward and inverse fast Fourier transform.
- Time-domain weighting by Hanning.
- Self-conjugate block multiplication in single or double precision.
- Block addition/subtraction and scaling in single or double precision.

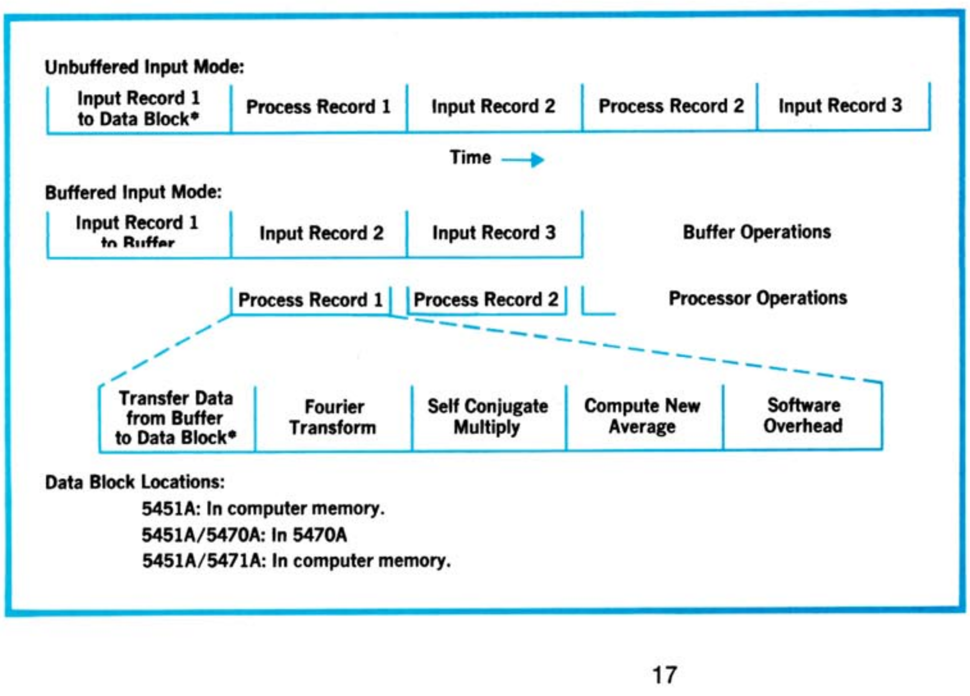

Fig. 4. *Timing diagrams for computation of power spectrum average. New buffered input mode is capability of 5451A Fourier Analyzer with or without fast-processor options. Parallel input-while-processing operation in buffered mode increases system efficiency.*

Multiple array operations are used to perform complete spectrum analysis functions such as:

- Auto and cross power spectrum average.
- Auto and cross correlation.
- Convolution.
- Transfer and coherence functions with the aid of division routines in the host computer.

The Where and How of Fourier Analyzers

Here are examples of how HP Fourier Analyzers are being used in various locations around the world:

Massachusetts. Used aboard a research ship for sea floor exploration in search of oil fields.

Canada. Underwater Acoustics. Extensive use of coherence and transfer function analysis to determine optimum sonar bubble shape.

Germany. Acoustic and vibration analysis of linear servo systems.

California. Civil Engineering. Used to study dynamic characteristics of large structures, such as resonant frequencies and damping factors, to determine how they will withstand earthquakes.

Sweden. Biomedical. Electromyography. Used in studying types of signals transmitted on various types of nerves for designing automatic prostheses.

Michigan. Vibration analysis on vehicle drive trains in production environment to determine correct/incorrect operation of gears and other parts.

Iowa. Vibration analysis on tractors to aid in design of drive train and frame components.

Belgium. Vibration analysis on machinery used in the application of emulsion to film to determine effect of vibrations on emulsion.

New York. Vibration analysis on cutting tools to determine wear characteristics.

France. Vibration studies of helicopters aimed at noise reduction.

Netherlands. Civil Engineering. Analysis of vibrations of tunnels, dikes and other large structures to determine dynamic characteristics.

Michigan. Vibration analysis of production power tools to determine wear and efficiency.

Michigan. Vibration analysis of axles and axle housings to improve design and for production testing.

Michigan. Vehicle crash studies. Used to design automobile components such as steering wheels.

Fast Fourier Processor

The 5470A Fast Fourier Processor is a digital processor designed to perform the Fourier analysis routines rapidly and at reasonable cost. As a peripheral to the 5451A system, the 5470A operates under functional control of the host computer which provides it data, microprograms, and execution command parameters.

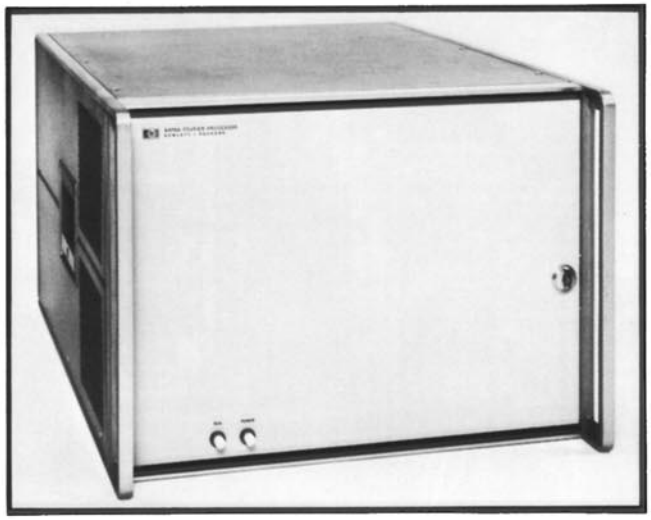

Fig. 5. *New Model 5470A Fast Fourier Processor used with 5451A Fourier Analyzer lowers time for 1024-point transform to 15 milliseconds. Processor can also add Fourier-transform capability to other computer systems.*

Nearly all data-array manipulations normally needed for spectrum analysis are performed in the Fast Fourier Processor. It has either a 4096-word data memory or an optional 8192-word data memory, and data blocks may range in size from 64 to 2048 words (4096 optional).

When a power spectrum average is being computed, data flow in the combined 5451A/5470A system is similar to that in the analyzer alone. After the buffer in the computer is filled with input data from the analog-to-digital converter, the data is transferred to the data memory in the 5470A. For each data record a time-to-frequency Fourier transform, a self-conjugate multiply, and an addition into the accumulation block are successively performed in the 5470A. While these operations are in progress the computer is free to load the next data record into the buffer section of its memory from the analog-to-digital converter, and send output to the display if time permits.

All block operations on data in the 5470A are controlled by microprograms from the computer. For each operation a microprogram from the software driver is loaded into the processor's function control memory and then executed. Each user command may call for multiple 5470A operations.

The Fast Fourier Processor is linked to the computer in the 5451A Fourier Analyzer by two I/O channels which use standard 16-bit TTL microcircuit interface cards. Since the interface cards are standard, the fast processor doesn't have to be used with a Fourier analyzer; it can add Fourier-analysis capability to other computer systems as well.

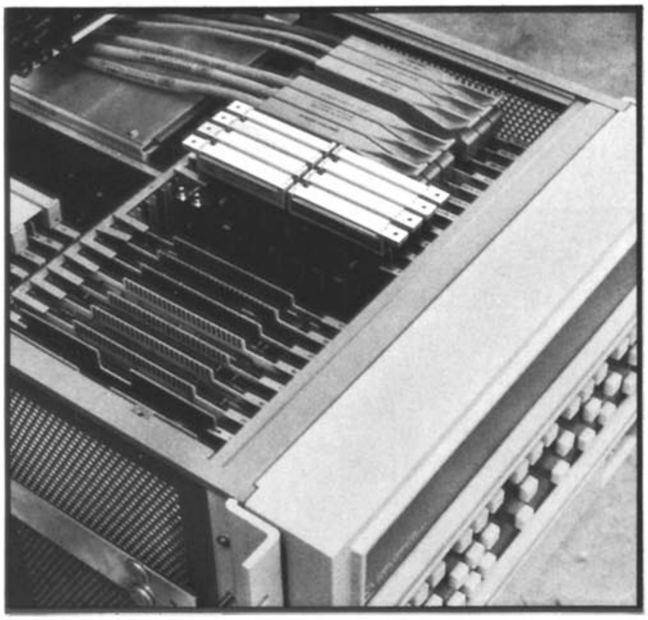

Fig. 6. *New Model 5471A Fast Fourier Transform Arithmetic Unit plugs into four I/O slots in HP computers. 1024-point transform time is 160 milliseconds.*

Fast Fourier Transform Arithmetic Unit

The 5471A Fast Fourier Transform Arithmetic Unit speeds up by a factor of 6 or more many of the operations required in Fourier analysis. The FFT Arithmetic Unit is a hardware-software package. The hardware consists of four printed-circuit boards with plug into any four contiguous I/O slots in a Hewlett-Packard 2100-series computer.

The software consists of a driver written in HP assembly language. The driver has been incorporated into the Fourier analyzer software so to the user the operation is the same as the standard analyzer except for the faster speed. Another assembly language driver allows the 5471A to be used with a stand-alone computer by those users who need Fourier arithmetic capability but don't want a complete Fourier analyzer system.

Data is stored in the computer, not in the arithmetic-unit hardware. The 5471A memory is simply a small scratchpad. Memory addressing and read/writing are done by software while the arithmetic operations are done by the 5471A hardware.

SPECIFICATIONS

HP Model 5451A
Fourier Analyzer

ANALOG INPUT
AMPLITUDE RANGE: 0.1 V to 10 V full scale.
RESOLUTION: 10 bits with sign.

DISPLAY UNIT
Data may be displayed on the 8 × 10 cm CRT display or output to a plotter or remote display.

AMPLITUDE SCALE: Data in memory is automatically scaled to give a maximum on-screen calibrated display.
LINEAR DISPLAY RANGE: ±4 divisions with scale factor ranging from 1×10^{-512} to $5 \times 10^{+512}$ in steps of 1, 2, 5, and 10.
LOG DISPLAY RANGE: 4 decades with a scale factor ranging from 0 to ±998 dB.

DIGITAL ACCURACY AND RESOLUTION
All calculations use floating point arithmetic on a block basis. Data overflow does not occur. Amplitude resolution is 1 part in 16,000 worst case.
DATA WORD SIZE: 16-bit real and 16-bit imaginary or 16-bit magnitude and 16-bit phase. 32 bits real is preserved for auto power spectra. Division, addition, and subtraction operations performed in 16 bits or 32 bits depending on data.
TRANSFORM ACCURACY: 0.1% of full scale worst case error during forward or inverse calculation.
TRANSFORM SIZE: From 64 to 4096 time points.
TRANSFORM SPEED: 1 s for 1024 time points.

SPECTRAL RESOLUTION
The element of spectral resolution is the frequency channel width —the maximum frequency divided by ½ the data block size.
MAXIMUM FREQUENCY: 25 kHz single channel; 10 kHz dual channel. Adjustable in steps of 1, 2.5, 5, and 10 down to 0.1 Hz.
DYNAMIC RANGE: 80 dB for voltage magnitude spectrum and auto power spectrum.

POWER SOURCE: 115/230 volts ±10%, 50/60 Hz. 1400 watts typical for basic system (excluding teleprinter).

ENVIRONMENTAL CONDITIONS: 0°C to 55°C.
Relative Humidity: To 95% at 40°C (104°F).

PRICE IN USA: 5451A System prices start at $35,000.

HP Model 5470A
Fast Fourier Processor

TRANSFORM BLOCK SIZE: 64 to 2048 real points standard. Up to 4096 points optional.

DATA MEMORY:
4096 16-bit words, optional 8K available.

SINGLE PRECISION OPERATIONS:
Fast Fourier transforms—forward and inverse.
Interval centered Hanning.
Self multiply; real, complex, and conjugate.
Cross multiply; real, complex, and conjugate.
Scale and add.

DOUBLE PRECISION OPERATIONS:
Self conjugate multiply.
Scale and add.

FOURIER TRANSFORM TIME (based on a 1024 real point block size): 15.2 ms.

ENVIRONMENTAL: 0°C to 55°C.

PRICE IN USA: 5470A prices start at $25,000.

HP Model 5471A
Fast Fourier Transform Arithmetic Unit

ARRAY SIZES
TRANSFORM BLOCK SIZE: 64 to 2048 points in powers of 2.
ADDITION BLOCK SIZE: 64 to 2048 points in powers of 2.

SINGLE PRECISION OPERATIONS
Forward and inverse fast Fourier transforms.
Interval centered Hanning.
Real, complex, and complex conjugate multiply.
Block scaling and block addition.

DOUBLE PRECISION OPERATIONS
Self complex conjugate multiply.
Block scaling and block addition.

FOURIER TRANSFORM TIME (based on 1024 word block size): 160 ms with 2100A Computer.

ENVIRONMENTAL: 0° to 55°C.

PRICE IN USA: 5471A prices start at $4500.

MANUFACTURING DIVISION: SANTA CLARA DIVISION
5301 Stevens Creek Boulevard
Santa Clara, California 95050

The 5471A must be microprogrammed for each type of arithmetic operation (i.e., Fourier transform, complex multiply, double-precision self-conjugate-complex multiply, Hanning, etc.). For each operation a microprogram from the software driver is loaded into the program memory on the 5471A program board and then the 5471A is ready to start.

The design philosophy of the 5471A is analogous to the concept of impedance matching for optimal power transfer. The 5471A matches times instead of impedance. It makes the time required for the hardware to perform its arithmetic tasks approximately equal to the time required for the software to output 16-bit data words to the 5471A, generate the next data addresses, and then input 16-bit data words from the 5471A to the computer. The arithmetic operations consume approximately 85% of the machine time required to do a Fourier transform. The 5471A hardware actually processes the data faster than the computer can calculate the next addresses. Thus the 5471A is able to speed up the transform time by a factor of 6 to 7.

Acknowledgments

The 5451A systems development has been under the capable leadership of Skip Ross, Ago Kiss, and Pete Roth. Chuck Hershkowitz has written most of the 5451A software.

The 5470A has had many contributors. Hans Nadig designed the arithmetic unit. Dick Grote had responsibility for the memory unit. The program unit was designed by Webb McKinney. Product design was handled by Chuck Lowe and technician assistance by Bill Katz. Dave Snyder has written the software to drive the 5470A.

5471A hardware and software development was assisted by Dolores Solorio.

Credit for getting the 5451A, 5470A, and 5471A into production must go to Walt Noble, our very capable production engineer.

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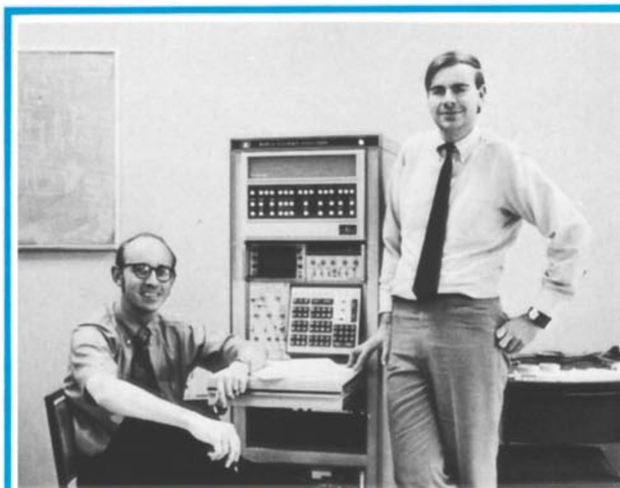

Stephan G. Cline

Steve Cline (right) is project leader for the 5471A FFT Arithmetic Unit. He's been working with Fourier Analyzers ever since he first came to HP in 1968. He wrote much of the software for the 5450A Fourier Analyzer and was involved in the design of the 5470A Fast Fourier Processor. Steve received his BS degree in electrical engineering from Michigan State University in 1967 and his MSEE from Stanford University in 1968. In his spare time, he expresses his concern for others in a tangible way by working in the Big Brother program. Golf is his main recreation, but he's also into model railroading and biking, both very useful, no doubt, in his Big Brother activities.

Norman D. Marschke

Norm Marschke, project leader for the 5470A Fast Fourier Processor, came to HP in 1964 soon after receiving his MS degree in electrical engineering from the University of Michigan. His BSEE degree is also from Michigan. At HP, Norm has helped design elements of the 5245 Counter system, and has served as project leader for the digital processor in the 5400A/5401A Multichannel Analyzers and for an analyzer/calculator interface. Norm says that a new home now dominates his spare time, but he always makes the annual migration to the mountains when ski season rolls around.

HEWLETT-PACKARD JOURNAL

***JUNE 1972** Volume 23 • Number 10*

Technical Information from the Laboratories of Hewlett-Packard Company, 1501 Page Mill Road, Palo Alto, California 94304 U.S.A.
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